INFLUENCE OF MODELLING AND NUMERICAL PARAMETERS ON THE PERFORMANCE OF A FLEXIBLE MBS FORMULATION

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SYNOPSIS

Recently, the authors have developed an efficient, robust, accurate and easy-to-implement method for the real-time analysis of rigid-flexible multibody systems. The flexible bodies are modelled by means of the floating frame of reference formulation, along with modal superposition of both static and dynamic modes. The dynamic modes to be considered for each flexible body must be decided by the analyst.
On the other hand, the co-rotational approach used to derive the inertia terms of the dynamic equations motivates that such terms depend on the discretization of the underlying finite element mesh. Therefore, the discretization size of the finite element model is another parameter to be selected by the analyst.
Furthermore, the value of two other parameters must be chosen: the penalty factor for the dynamic equations, and the time-step size for the fixed single step numerical integrator.
This paper studies the influence of the four parameters on the accuracy and efficiency of the abovementioned method, along with their relative dependence. To this end, a sweeping of the space generated by the parameters is carried out for a flexible system, and the corresponding results are analyzed in terms of accuracy and efficiency. In order to have a reference for comparison, the system is also solved through the nonlinear module of a finite element analysis commercial code.

NOMENCLATURE

p1, **p2**: points (natural coordinates) used to model the pinned-free beam.
$p2_x$, $p2_y$: *x*- and *y*-coordinate of point **p2**.
v1, **v2**: unit vectors (natural coordinates) used to model the pinned-free beam.

$v1_x$, $v1_y$: x- and y-component of unit vector **v1**.

$v2_x$, $v2_y$: x- and y-component of unit vector **v2**.

$\mathbf{\Phi}$: static bending mode of the pinned-free beam.

η: amplitude of static bending mode $\mathbf{\Phi}$.

n: number of the first dynamic bending modes considered.

$\mathbf{\Psi_1}, \mathbf{\Psi_2}, ..., \mathbf{\Psi_n}$: n first dynamic bending modes of the pinned-free beam.

$\xi_1, \xi_2, ..., \xi_n$: amplitudes of the n first dynamic bending modes $\mathbf{\Psi_1}, \mathbf{\Psi_2}, ..., \mathbf{\Psi_n}$.

q: vector of problem variables.

m: number of finite elements used for the discretization of the pinned-free beam.

z_i : history of a certain magnitude.

z_i^* : history of a certain magnitude for the reference simulation.

$|z|_{\max}$: maximum absolute value of a certain magnitude during the simulation.

α: penalty factor for the augmented Lagrangian dynamic formulation.

Δt: fixed time-step selected for the numerical integration.

1 INTRODUCTION

During the last years, the authors have developed an efficient, robust, accurate and easy-to-implement method for the real-time analysis of rigid-flexible multibody systems[1,2]. The method employs natural coordinates for the modelling[3], applies the co-rotational approach[4] to derive the inertia terms of the flexible bodies, establishes the equations of motion through an index-3 augmented Lagrangian formulation with projections in velocities and accelerations[5], and carries out the numerical integration by means of the implicit, single step trapezoidal rule[6]. The kinematics of the flexible bodies is introduced through the floating frame of reference approach[7], along with modal superposition to describe the corresponding local deformations[8], carried out by means of both static and dynamic modes defined with respect to a tangent frame[9].

When a certain multibody system containing flexible bodies is to be studied through the described method, four kinds of parameters are left to the analyst decision:

a) The dynamic modes to be considered for each flexible body. Once the modelling in natural coordinates of the whole multibody system has been carried out, the static modes for each flexible body are automatically established[1] (some of them can be neglected, if desired, by imposing the corresponding constraint equation of null amplitude). However, the dynamic modes, which have the role of improving the representation of the deformation field given by the static modes, can arbitrarily be included in the model. Decision about how many and which dynamic modes to consider must be taken by the analyst. As demonstrated in previous works[10,11], both the accuracy and the efficiency of the simulation will be strongly influenced by this choice.

b) In a general approach, a finite element (FE) model of each flexible body is also prepared. Such model serves, in a pre-processing stage, to obtain the static and dynamic modes, as well as the mass and stiffness matrices of the finite element method and, in a post-processing stage carried out at each time-step, to work out the values of elastic strains, stresses, displacements and efforts. Hence, the way in which the body is discretized becomes relevant, since it is expected to affect both to the accuracy and the efficiency of the simulation. For flexible bodies of simple geometry, like straight and uniform beams, the analytical form of the modes is available and, therefore, the described pre- and post-processing stages are not needed, and the simulation behaviour no longer depends on the FE mesh. However, if the authors' method is used, the FE model still appears in the formulation. The reason is that, when the co-rotational approach is introduced, the inertia terms of the dynamic equations for each flexible body are obtained as products of several matrices which depend on the FE model[1,2]. Therefore, either if the analytical modes are available or not, the adopted FE discretization affects to the performance of the simulation. This is the second decision left to the analyst.

c) In the proposed method, the equations of motion are established by means of an index-3 augmented Lagrangian formulation, which requires a penalty factor to amplify the internal forces caused by constraint violations. Such factor is crucial for the simulation stability, and constitutes the third decision to be taken by the analyst.

d) Since the described method is targeted to achieve real-time performance, the fixed single step trapezoidal rule is used. Therefore, the fixed time-step for the numerical integration must be selected: this is the fourth decision for the analyst.

Choices (a) and (b) can be referred to as the *modelling* parameters, since they deal with the modelling of each flexible body, while (c) and (d) may be called the *numerical* parameters, as they are related to the dynamic formulation and integration procedure of the whole multibody system.

This paper aims to study the influence that the four mentioned parameters have on both the efficiency and the accuracy of the proposed method, and to search for relationships among such four parameters. To achieve these objectives, a sweeping of the space generated by the two modelling parameters is carried out for a flexible system, and the two numerical parameters are adjusted for each combination. The results are analyzed in terms of accuracy and efficiency. In order to have a reference for comparison, the example is also solved through the nonlinear module of a finite element analysis (FEA) commercial code.

The remaining of the paper is organized as follows: Section 2 shows the flexible system to be analyzed, along with its modelling with both the proposed and the FE method; Section 3 explains the characteristics of the motion undergone by the system, the criteria to generate the multiple simulations executed, the magnitudes to be recorded, and the way to determine the error incurred by each simulation; Section 4 presents the results obtained for all the simulations, which are discussed in Section 5; finally, Section 6 summarizes the conclusions of the work.

2 THE EXAMPLE

The flexible system to be analyzed, shown in Figure 1a, consists of a beam pinned at one end to the ground, which starts from the rest and undergoes the bang-bang torque depicted in Figure 1b. Gravity effects are neglected. Physical properties of the beam are: mass density 8000 Kg/m^3, modulus of elasticity $2x10^{11}$ N/m^2, length 1.5 m, cross-sectional area 10^{-4} m^2, moment of inertia 10^{-10} m^4.

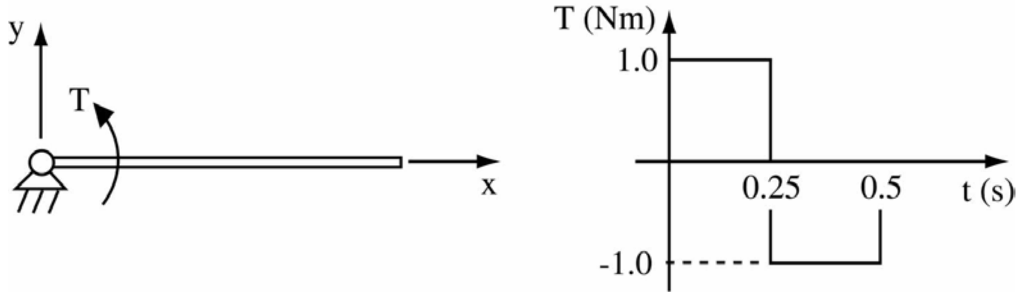


Fig. 1 a) pinned-free beam; b) bang-bang torque.

Following the method developed by the authors, the modelling of the beam has been carried out as illustrated in Figure 2. At the pinned end, point **p1** and unit vectors **v1** and **v2** have been defined, thus constituting the local reference frame of the body. In this case, point **p1** is fixed. At the free end, point **p2** has been defined. The local displacement of point **p2** in **v2**-direction activates static bending mode **Φ**. Its local displacement in **v1**-direction has been prevented through a constraint equation, so as to avoid the appearance of the corresponding axial mode, not relevant in this example. To better represent the deformed configuration of the beam, as many dynamic modes as desired can be considered: they are the natural modes of vibration of the beam with fixed boundaries (points **p1** and **p2**, and unit vectors **v1** and **v2**), which means that, for their calculation, left end must be clamped and right end must be pinned. Figure 2 shows the two first dynamic modes, $\boldsymbol{\Psi}_1$ and $\boldsymbol{\Psi}_2$.

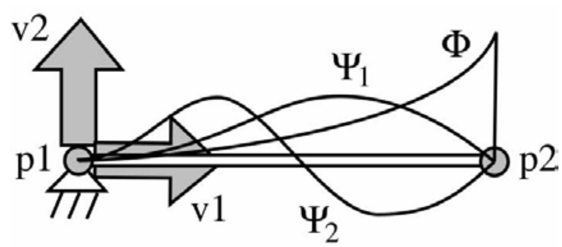


Fig. 2 Modelling of the flexible pinned-free beam with the authors' method.

Then, if a certain number *n* of dynamic modes is chosen for the modelling, the vector of problem variables results,

$$\mathbf{q}^{\mathrm{t}} = \left\{ v1_x \quad v1_y \quad v2_x \quad v2_y \quad \eta \quad \xi_1 \quad \xi_2 \quad \cdots \quad \xi_n \quad p2_x \quad p2_y \right\} \tag{1}$$

where η is the amplitude of the static deformation mode **Φ**, and ξ_1, ξ_2, ..., ξ_n are the amplitudes of the dynamic modes considered $\boldsymbol{\Psi}_1$, $\boldsymbol{\Psi}_2$, ..., $\boldsymbol{\Psi}_n$. Therefore, the total number of variables is 7+*n*, with only 2+*n* independent. The analytical forms of both the static and dynamic modes have been used.

For the underlying FE model of the beam, a mesh of *m* two-dimensional beam elements (BEAM2D) has been generated. All the elements are identical, with nodes of three degrees of freedom: two displacements in the plane of the beam and the corresponding slope. As the node placed at the pinned end of the beam can only experiment rotation, the total number of variables rises to $3m+1$.

3 THE ANALYSIS

The motion of the flexible beam undergoing the described torque is simulated for 2 s. Simulations are run with a number of dynamic modes n going from 0 to 4, and a number of beam elements m ranging from 2^1 to 2^6. The penalty factor is initially adjusted to 10^9, and increased only when bad results are obtained. The time-step is set to 1 ms; in case that the simulation fails, the time-step is reduced until good behaviour is achieved.

For each simulation, the following results are recorded: a) CPU-time required; b) history of the y-coordinate of the free end of the beam; c) history of the bending moment at the middle section of the beam. In order to have a reference for comparison, so as to evaluate the quality of the solution obtained at each simulation, the problem has also been solved through the nonlinear module of FEA commercial code COSMOS/M 2.8, using a discretization of 2^6=64 elements.

Two error values have been obtained for each simulation: a displacement error and a bending moment error. In both cases, the error has been calculated as follows. The history of the corresponding magnitude has been recorded at every 1 cs for both the simulation of reference and the simulation being evaluated. Then, the error is obtained as,

$$error = \left(\frac{1}{201} \frac{1}{|z|_{\max}} \sum_{i=0}^{200} \left| z_i - z_i^* \right| \right) \times 100 \tag{2}$$

where 201 is the number of values considered (steps of 1 cs during 2 s of simulation), z_i represents the history of the corresponding magnitude (y-coordinate of the free end of the beam, or bending moment at the middle section of the beam) for the current simulation, z_i^* is the same for the reference simulation, and $|z|_{\max}$ is the maximum absolute value of the magnitude during the simulation. The resulting errors have the form of percentages.

4 RESULTS

Table 1 shows the obtained results for all the simulations performed. Remember that n is the number of dynamic modes, m is the number of beam elements, α is the penalty factor, and Δt is the fixed time-step selected for the numerical integration. The symbol "---" means that the simulation failed with such a combination of dynamic modes and discretization size. The simulation which produces the most accurate results has been boldfaced. The CPU-times reported have been obtained on a Pentium III @ 900 MHz.

Table 1 CPU-time and errors for all the simulations performed.

#sim	n	m	α	Δt (s)	CPU-time (s)	Error in displacement (%)	Error in bending moment (%)
1	0	2	10^9	10^{-3}	0.15	8.86	97.68
2		4	10^9	10^{-3}	0.16	9.50	74.59
3		8	10^9	10^{-3}	0.25	11.33	81.23
4		16	10^9	10^{-3}	0.56	11.63	81.93
5		32	10^9	10^{-3}	1.71	11.58	81.50
6		64	10^9	10^{-3}	7.19	11.57	81.35
7	1	2	---	---	---	---	---
8		4	10^9	10^{-3}	0.22	11.17	61.94
9		8	10^9	10^{-3}	0.29	7.48	48.62
10		16	10^9	10^{-3}	0.83	3.63	25.59
11		**32**	$\mathbf{10^9}$	$\mathbf{10^{-3}}$	**2.40**	**2.10**	**19.95**
12		64	10^9	10^{-3}	8.32	2.24	20.32
13	2	2	---	---	---	---	---
14		4	---	---	---	---	---
15		8	10^9	10^{-3}	0.49	8.10	46.10
16		16	10^9	10^{-3}	1.12	6.41	37.20
17		32	10^9	10^{-3}	2.99	3.68	25.26
18		64	10^9	10^{-3}	11.07	3.58	25.06
19	3	2	---	---	---	---	---
20		4	---	---	---	---	---
21		8	---	---	---	---	---
22		16	10^9	10^{-4}	8.00	4.25	23.40
23		32	10^9	10^{-4}	22.77	8.91	51.43
24		64	10^9	10^{-4}	80.88	6.48	34.72
25	4	2	---	---	---	---	---
26		4	---	---	---	---	---
27		8	---	---	---	---	---
28		16	10^{10}	10^{-4}	9.22	7.44	35.86
29		32	10^9	10^{-4}	25.56	3.09	20.23
30		64	10^9	10^{-4}	93.68	5.22	28.76

In order to provide the reader with a more visual presentation of the results, the CPU-time and errors for all the simulations performed are also given in Figure 3. CPU-times of 100, and error values of 20 for displacements and 100 for bending moments have been assigned to those simulations which failed (symbol "---" in Table 1), so that plots are not distorted.

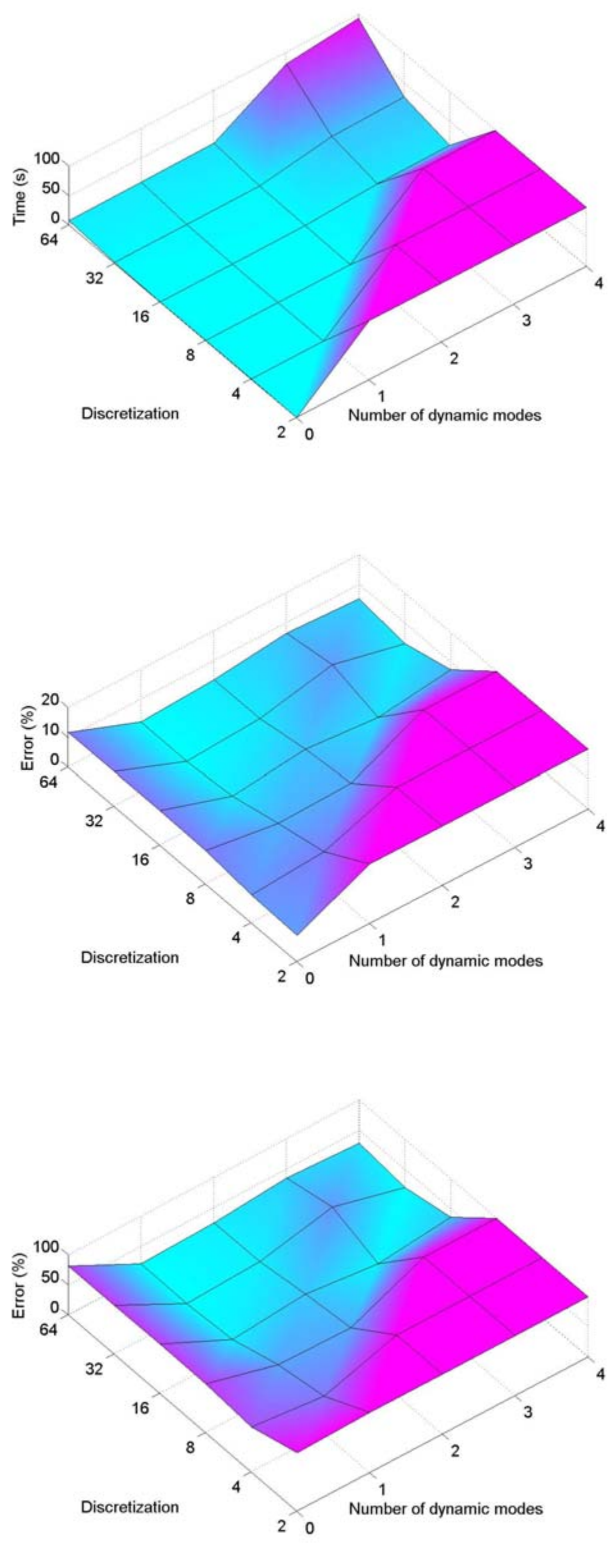
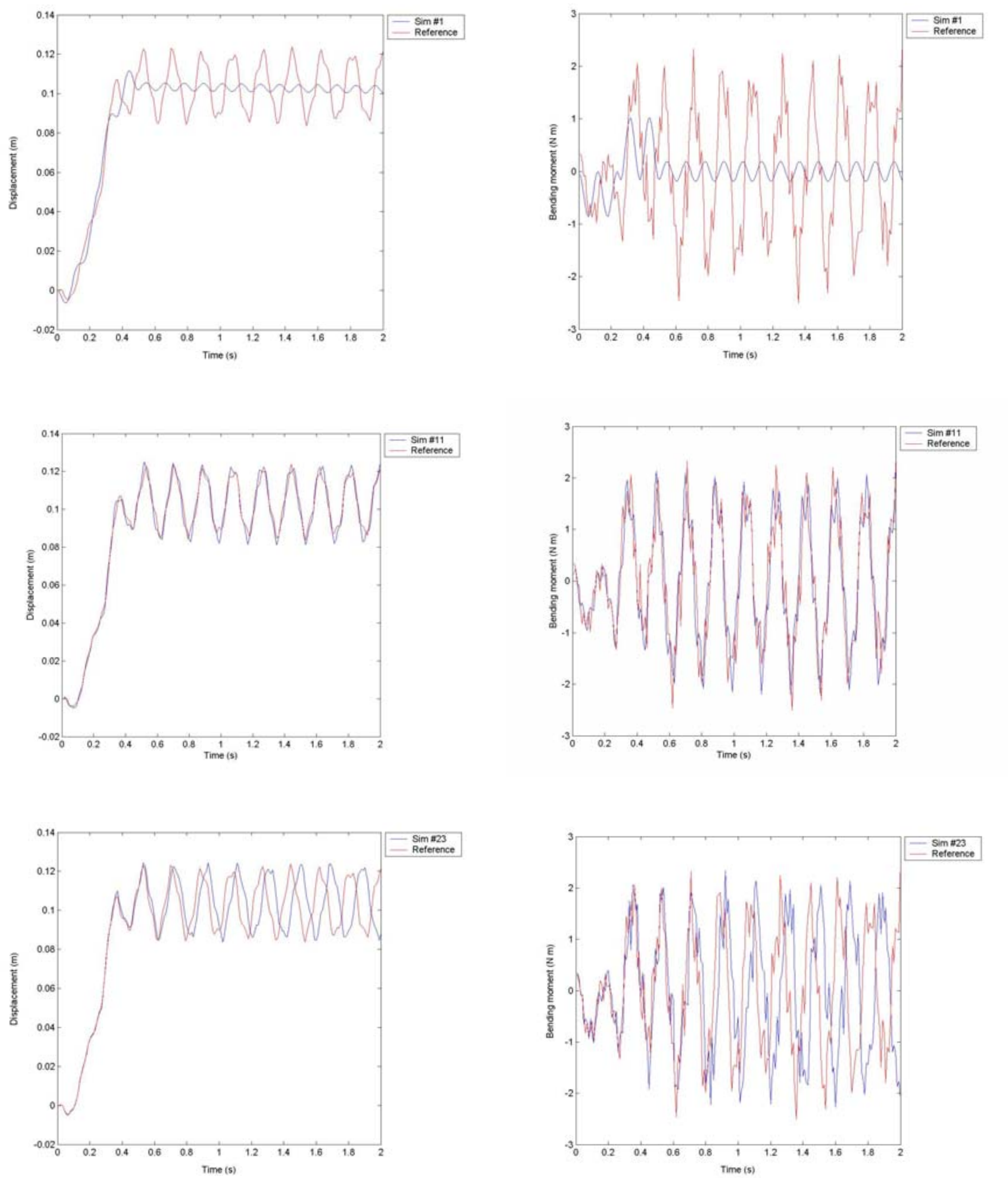


Fig. 3 Results: a) CPU-time; b) displacement error; c) bending moment error.

Both errors (displacement and bending moment) have been reduced to single figures, according to Equation (2), in an attempt of condensing the information and making easier its interpretation. In order to show the correlation between the error values presented in Table 1, and the actual discrepancies of the simulations with respect to the reference, plots comparing the histories of displacement and bending moment for simulations 1, 11, 23 and 30 along with the corresponding histories for the reference simulation are depicted in Figure 4.



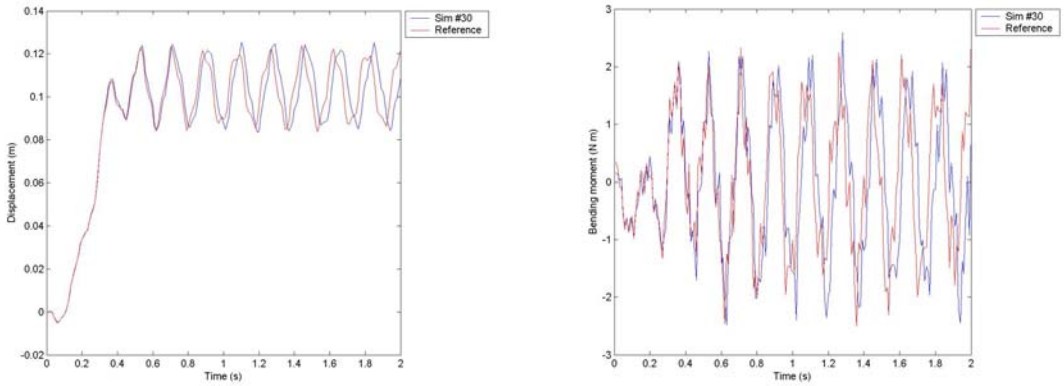


Fig. 4: a) Errors in simul. #1 (*n*=0, *m*=2): displ. (8.86%), bend. moment (97.68%);
b) Errors in simul. #11 (*n*=1, *m*=32): displ. (2.10%), bend. moment (19.95%);
c) Errors in simul. #23 (*n*=3, *m*=32): displ. (8.91%), bend. moment (51.43%);
d) Errors in simul. #30 (*n*=4, *m*=64): displ. (5.22%), bend. moment (28.76%).

From the presented plots, it can be seen that, if the error is determined as proposed in Equation (2), an error in displacement of 8% is a great error, while a value of 2% means very good accuracy. On the other hand, an error in bending moment of 100% represents a large error, and 20% indicates excellent agreement with the reference. Although the scales of both errors are different, as are the mean values of each kind of results, their trend is, in general, the same.

5 DISCUSSION

At the view of the results presented in the previous Section, it is clear that consideration of more dynamic modes does not necessarily leads to more accurate results, but always to less efficient simulations. In the example, the most accurate results are obtained with only one dynamic mode, and the corresponding efficiency is high in the context of all the executed simulations. This means that an optimum number of dynamic modes exists for a certain analysis.

Regarding the discretization, it can be affirmed that a maximum mesh size cannot be exceeded based on the highest dynamic mode considered. Once under such maximum, more accurate results are obtained for more refined meshes until a certain value; further refinement does not lead to any improvement. On the other hand, the efficiency decreases as the mesh is refined. Therefore, it comes out that the required discretization depends on the number of dynamic modes, and that there is also an optimum size for the underlying FE mesh. It must be pointed out that the exponential increment in CPU-time reported in Table 1 as the number of finite elements rises, is due to products of matrices whose size depends on the mesh size, needed to build up the inertia terms. Such products can be done more efficiently if the sparse structure of the arrays is accounted for, so attenuating the pronounced growth of the CPU-times with respect to the discretization size.

Going now to the numerical parameters of the method under study, it seems that an increment of the penalty value can be worthy only for cases of insufficient FE discretization. On the other hand, the time-step size needed to perform the numerical integration shows to be related to the highest dynamic mode included in the modelling: higher dynamic modes imply smaller time-step sizes and, consequently, lower efficiency.

From what has been said, it can be concluded that only one parameter is independent: the number of dynamic modes. The other three parameters –discretization size, penalty factor and time-step size– can be established as functions of the number of dynamic modes. Moreover, there is an optimum value of the number of dynamic modes for a certain problem. The reason is that the motion of the body is properly captured with such optimum number of dynamic modes, so that the inclusion of additional modes only leads to the appearance of higher frequencies in the solution, which in turn hinder the numerical integration process, thus producing higher errors. Therefore, a method to determine how many and which dynamic modes must be considered for a certain analysis is crucial to develop models which can be run on real-time with the proposed formulation. Of course, the iterative process will always be available, and can be an option for some applications.

In the field of structural dynamics, the optimum number of dynamic modes depends on the physical properties of the body and the frequency content of the applied forces, and both of them can be analyzed before the simulation is carried out. However, in flexible multibody dynamics, joint and inertia forces, which cannot be analyzed a priori, are of key relevance; they depend on the motion undergone by the body, which is unknown until a simulation is performed. Therefore, it seems that making an initial estimation of the optimum number of dynamic modes in flexible multibody dynamics won't be an easy task, and that, at least, a previous rigid-body simulation will be required in order to get some insight into the form of both the joint and inertia forces.

Table 2 MPFs for the example.

# dyn. mode	MPF (%)
1	100
2	3.46
3	0.77
4	0.17
5	0.06
6	0.02
7	0.01

This is exactly what is proposed in the modal participation factor (MPF) method, which has been successfully employed recently[11,12] to estimate the dynamic modes that must be included in the model of a flexible multibody system. Such method has been applied in the paper to the studied example, so as to correlate it with the obtained results. To this end, rigid-body simulation of the system has been carried out, and the most critical position identified. The forces acting upon the body in such position have been recorded, and a static analysis of the body, now considered as a structure, has been conducted. For this purpose, the FE model of 64 elements, which had been served for comparison so far, has been used, and the first 16 dynamic modes have been obtained. Both the stiffness matrix and the applied forces have been projected to the modal space, and the corresponding modal amplitudes derived from the static equilibrium equation. Table 2 shows the MPFs obtained for the first 7 dynamic modes. The MPFs of the remaining modes were even smaller.

From the results presented in Table 2, it is clear that the first dynamic mode prevails, and that only the second is above the commonly accepted limit of 1% share. Therefore, the MPF method indicates that the optimal selection consists in just taking the first dynamic mode, or perhaps the first and the second ones, which is in good agreement with the results previously obtained in the paper. Consequently, the MPF method can be considered as a good candidate to provide an initial estimation of the optimum number of dynamic modes for each flexible body. Once such decision is taken, the automatic tuning of the other three parameters to their optimal values (discretization size for each flexible body, penalty factor and time-step size), seems to be relatively easier; the development of some method for this purpose will be addressed in the future.

6 CONCLUSIONS

Based on the previously exposed results and discussion, the conclusions can be drawn as follows:

a) The authors have recently proposed an efficient, robust, accurate and easy-to-implement method for the real-time dynamics of rigid-flexible multibody systems, based on the floating frame of reference formulation, with both static and dynamic modes.

b) When applying such method, the analyst must decide on the value of four parameters: two modelling parameters –number of dynamic modes and discretization size of the underlying FE mesh for each flexible body–, and two numerical parameters –penalty factor for the dynamic equations and fixed time-step size of the numerical integration–.

c) The four mentioned parameters are not independent: a certain value of the number of dynamic modes will ask for corresponding optimum values of the other three parameters.

d) An optimum number of dynamic modes exists for a certain problem, which leads to the best results in terms of accuracy.

e) The modal participation factor method can be used to provide an initial estimation of such optimum number of dynamic modes.

f) A method to automatically obtain the optimum values of the remaining three parameters once the number of dynamic modes has been decided is left for future development.

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