9th CONFERENCE
on
DYNAMICAL SYSTEMS
THEORY AND APPLICATIONS
December 17-20, 2007. Łódź, Poland

STABLE NUMERICAL DIFFERENTIATION IN THE CONTEXT OF THE KINEMATIC AND DYNAMIC ANALYSIS OF BIOMECHANICAL SYSTEMS

Francisco Javier Alonso, Javier Cuadrado, José María Del Castillo

Abstract: Kinematic and dynamic analysis of biomechanical systems is corrupted by numerous sources of error that reduce its usefulness. The main source of error is the inaccuracy of velocities and accelerations derived from experimentally measured displacements of markers placed on the skin of joints. This error is mainly due to the amplification of high-frequency low-amplitude noise introduced by the motion capture system when the raw displacement signals are differentiated. The raw data differentiation problem is well known to be ill-posed in the sense that a small error in position data can induce a large error in the approximate derivatives. In this work we propose the use of a numerical differentiation scheme based on the Newmark integration method, widely applied in structural dynamics. This method introduces numerical dissipation so as to damp out the spurious high frequency responses, and provides the smoothing and differentiation of the displacement signal in a single step. Examples show that the technique is well suited for the task of smoothing and differentiation of displacement signals in order to obtain accurate higher derivatives in a simple and systematic way.

1. Introduction

The inverse dynamic analysis (IDA) of biomechanical systems uses kinematic and anthropometric data to calculate net joint reaction forces and net driver moments during a physical activity or motion. In biomechanical studies, IDA is corrupted by numerous sources of error that reduce its usefulness. The main sources of error have recently been pointed out by Hatze [1], and include the inaccuracy of velocities and accelerations derived from experimentally measured displacements of markers placed on the skin of joints. This error is due to the amplification of high-frequency low-amplitude noise introduced by the motion capture system when the raw displacement signals are differentiated [2–5]. To avoid this phenomenon, it is necessary to filter or to smooth the displacement signal prior to differentiation.

The filtering of displacement signals to obtain noiseless velocities and accelerations has been extensively treated in the literature. Traditional filtering techniques include digital Butterworth filters, splines, and filters based on spectral analysis [2–5]. Nonetheless, traditional filtering methods are not suited to smoothing nonstationary signals. This drawback is particularly acute in biomechanical analyses since physical activity involves impact-like floor reaction forces [2]. In order to filter non-stationary signals, advanced filtering techniques such as discrete wavelet transforms [6], and singular spectrum analysis (SSA) [7] have been used. Nonetheless, the drawback in these cases is the complexity of devising an automatic and systematic procedure. A mother wavelet function must be selected when using discrete wavelet transforms, and the window length and reconstruction parameters must be chosen when using the SSA.

The goal of this paper is to demonstrate the advantages of smoothing methods based on Newmark's integration techniques. One of these advantages is that the method performs the smoothing and differentiation of the displacement signal in a single step. To test its performance, the Newmark method will be applied to two different signals, stationary and non-stationary, acquired with our own motion capture system.

2. The Newmark method

The Newmark method is a single-step integration formula. The method constitutes a special category of finite difference methods that have been widely used in solving the multi-DOF second-order differential equations that appear in structural dynamics.

The state vector of the system at a time $t_{n+1} = t_n + h$ is deduced from the already-known state vector at time t_n, through a Taylor expansion of the displacements and velocities [8,9]. The following are the two basic equations proposed by Newmark [8] for determining displacements and velocities of the structure at time t_{n+1}

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h\left[(1-\gamma)\ddot{\mathbf{q}}_n + \gamma\ddot{\mathbf{q}}_{n+1}\right]. \tag{1}$$

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_n + h^2\left[(\frac{1}{2}-\beta)\ddot{\mathbf{q}}_n + \beta\ddot{\mathbf{q}}_{n+1}\right]. \tag{2}$$

The constants γ and β are parameters associated with the quadrature scheme. The parameter β denotes the variation of the acceleration during the incremental time step $h = t_{n+1} - t_n$. The choice of β implies different schemes of interpolation for the acceleration over a time step. The value $\beta = 0$ indicates a scheme equivalent to the central difference method, $\beta = 1/4$ is a constant average acceleration method, and the value $\beta = 1/6$ is a linear acceleration method. The parameter

γ relates to the numerical damping introduced by discretization in the time domain. For the case with $\gamma < 1/2$, there exists some negative numerical damping while, for $\gamma > 1/2$, positive numerical damping will occur [8,9]. The method has been demonstrated to be unconditionally stable when:

$$\gamma \geq \frac{1}{2}. \tag{3}$$

$$\beta \geq \frac{1}{4}(0.5 + \gamma)^2. \tag{4}$$

From equations (1-2), the accelerations and velocities of the structure at time step t_{n+1} can be solved as

$$\ddot{\mathbf{q}}_{n+1} = \frac{1}{\beta h^2}(\mathbf{q}_{n+1} - \mathbf{q}_n) - \frac{1}{\beta h}\dot{\mathbf{q}}_n - \left(\frac{1}{2\beta} - 1\right)\ddot{\mathbf{q}}_n. \tag{5}$$

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h(1-\gamma)\ddot{\mathbf{q}}_n + h\gamma\ddot{\mathbf{q}}_{n+1}. \tag{6}$$

3. Results

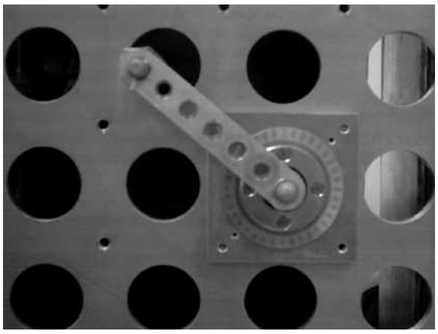
Two different signals will be differentiated and smoothed using the method described in the previous section.

3.1. Signal 1: Stationary sinusoidal signal

A reflective marker was attached to a uniformly rotating crank (Fig. 1a). The marker motion was captured for 8 s at a frequency of 100 Hz using three infrared cameras (Qualisys Medical AB). The vertical component of this motion (Fig. 1b) was smoothed and double differentiated using the Newmark method implemented in Matlab (The Mathworks Inc.).

The signal-to-noise ratio is very large in this example. Nonetheless, this small noise heavily contaminates the numerically obtained acceleration signal, and it is here, in the acceleration, where the efficiency of the method can be tested. The acceleration obtained using the original raw signal (Fig. 2a) can be compared with the acceleration obtained using the Newmark method (Fig. 2b) and the Butterworth filter (Fig. 2c).

(a) (b)



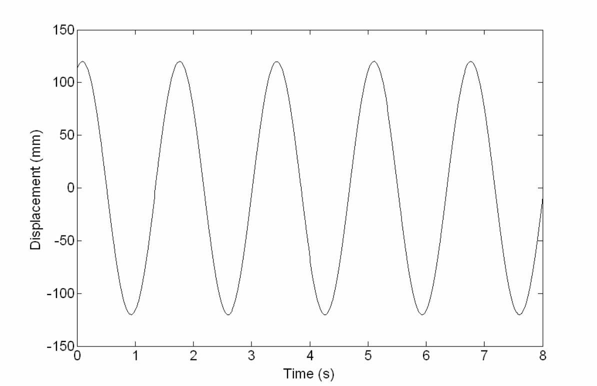


Fig. 1. (a) Experimental layout. (b) Noisy vertical displacement signal acquired.

It is clear that smoothing or filtering is absolutely mandatory in order to obtain meaningful acceleration signals. Moreover, the Newmark scheme is superior to Butterworth filtering. This statement is supported in the results shown in Fig. 2. Several γ and β parameters in the stability region (Eqs. (3-4)) were chosen, and it was found that for $\gamma = 3.5$ and $\beta = 10/4(0.5+\gamma)^2 = 40$ good results were obtained (see Fig. 2b). The original displacement signal was also passed through a second-order Butterworth filter with a 4 Hz cut-off frequency, and the result was differentiated twice using first-order finite differences to obtain acceleration. The superiority of Newmark method is particularly relevant in the elimination of the so-called end-point errors. Methods based on signal extension have been proposed to reduce end-point error [5]. No extension is necessary in the case of the Newmark scheme. Errors due to this phenomenon are negligible (Fig. 2b). The errors in terms of root mean square error (RMSE) with respect to the reference acceleration signal obtained with SSA [7] are 214.58 mm/s^2 for the Newmark method and 2191.83 mm/s^2 for the Butterworth filter, respectively

3.2. Signal 2: Slider motion

A vertical slider was moved by hand to obtain a non-stationary mono-dimensional motion of biomechanical origin. A subject was asked to move the slider randomly with fast upward and downward movements. The vertical position (Fig. 3) was acquired with the use of a marker attached to the slider and the Qualisys camera system (Qualisys Medical AB).

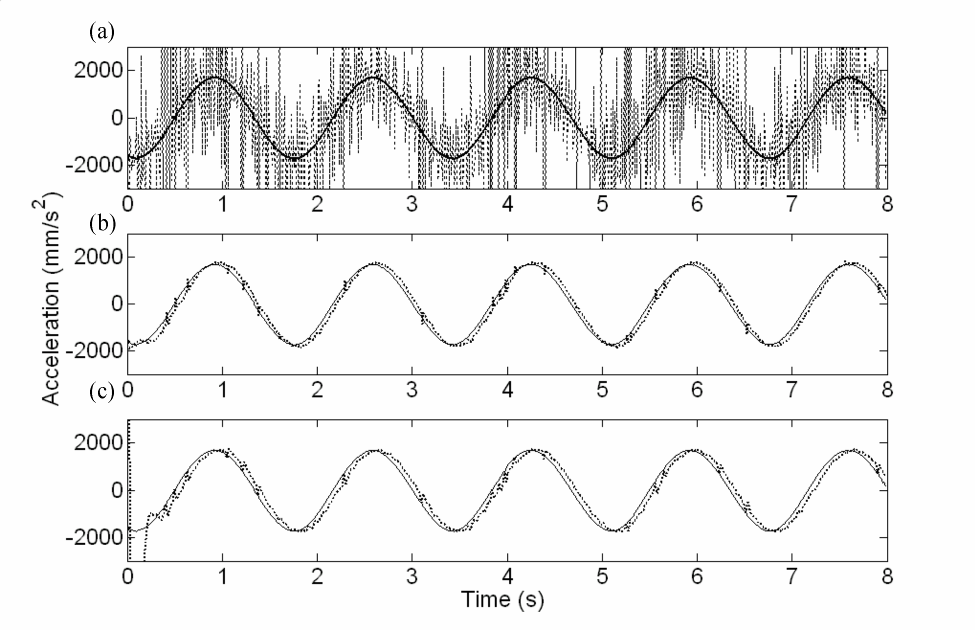


Fig. 2. (a) Acceleration calculated from Signal 1 original displacement data (dashed line) and acceleration calculated using SSA [7] (continuous line). (b) Acceleration calculated using Newmark method (dotted line). (c) Acceleration calculated from Signal 1 after having been passed through a 4 Hz cut-off frequency Butterworth filter (dotted line).

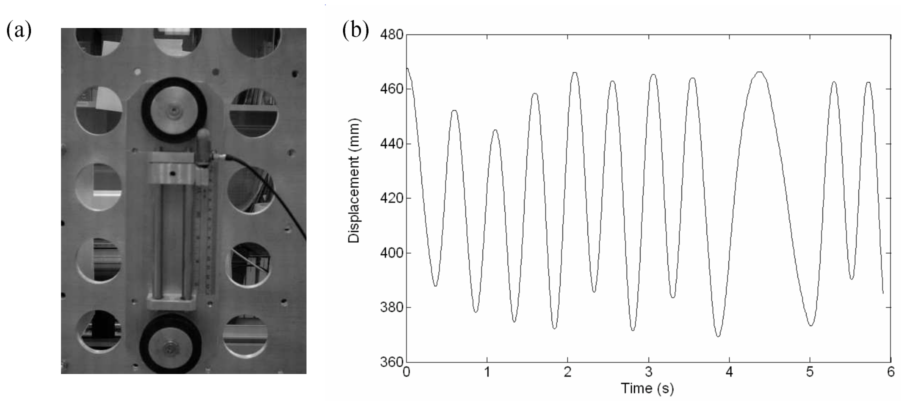


Fig. 3. (a) Experimental layout. (b) Noisy vertical displacement signal acquired.

The second derivative of this record, or of the record obtained after smoothing, is compared to the acceleration obtained directly from an accelerometer attached to the slider. Displacement and acceleration were sampled at 200 Hz during 5.90 s, obtaining records of 1182 elements each.

We performed a double differentiation using the Newmark method with $\gamma = 3.5$ and $\beta = 10/4(0.5+\gamma)^2 = 40$ (see Fig. 4b). The original displacement signal was also passed through a second-order Butterworth filter with a 4 Hz cut-off frequency, and the result was differentiated twice to obtain acceleration (Fig. 4c). These acceleration signals were compared to the accelerometer output in Fig. 4, where the acceleration obtained from the raw data, from the two reconstructed signals using Newmark or Butterworth filters, and directly from the accelerometer, are shown. The errors in terms of RMSE are 1.82 m/s2 for the Newmark method, 29.54 m/s2 for the Butterworth filter and 1.47 m/s2 using SSA [7] respectively.

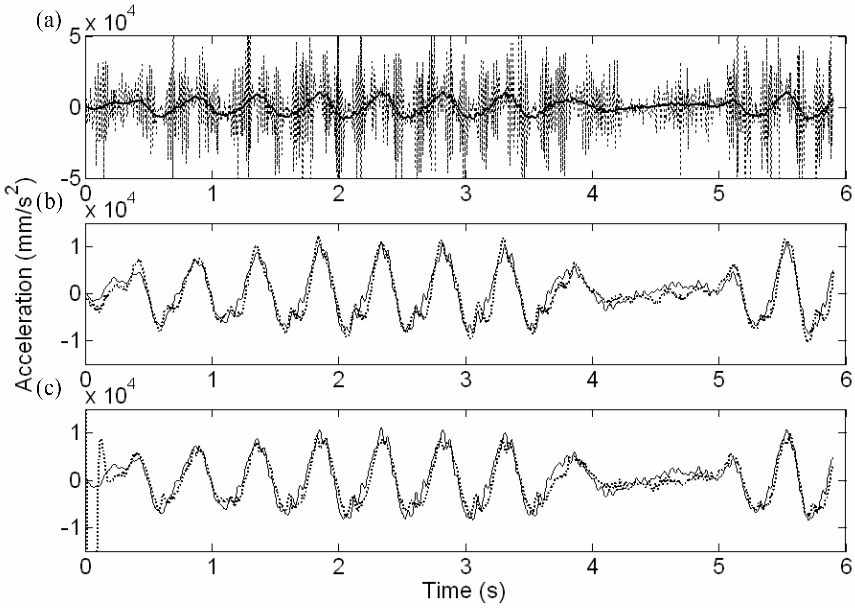


Fig. 4. (a) Acceleration calculated from Signal 2 original displacement data (dashed line) and acceleration measured by the accelerometer (continuous line). (b) Acceleration calculated using the Newmark method (dotted line). (c) Acceleration calculated from Signal 2 after having been passed through a 4 Hz cut-off frequency Butterworth filter (dotted line).

3. Discussion

The results of our study show that the Newmark method produces similar results that filtering techniques traditionally used in biomechanical analysis such us digital Butterworth filter and advanced smoothing techniques such us SSA. Our work uses the numerical dissipation of the Newmark method so as to damp out the spurious high frequency amplification in the differentiation process. This fact is key in the success of the method when it comes to extracting the latent trend in the signal from the random high-frequency noise inherent to the motion capture system. Moreover the Newmark method provides the smoothing and differentiation of the displacement signal in a single step. One of the main advantages of the method is the fact that the algorithm requires selection of just two parameters, namely γ and β, in the stability region (Eqs. (3-4)). It has also been shown that the method works properly on both stationary and non-stationary signals. Moreover, the method can be very easily programmed as a stand-alone automatic algorithm.

As drawbacks, it can be mentioned the fact that no fixed, objective rules for selecting the parameters γ and β have been presented. Figure 5 shows the evolution of RMSE with γ and β parameters for the two studied signals. The error is dramatically amplified for very low values of parameter β, and for low values of γ in the non-stationary case.

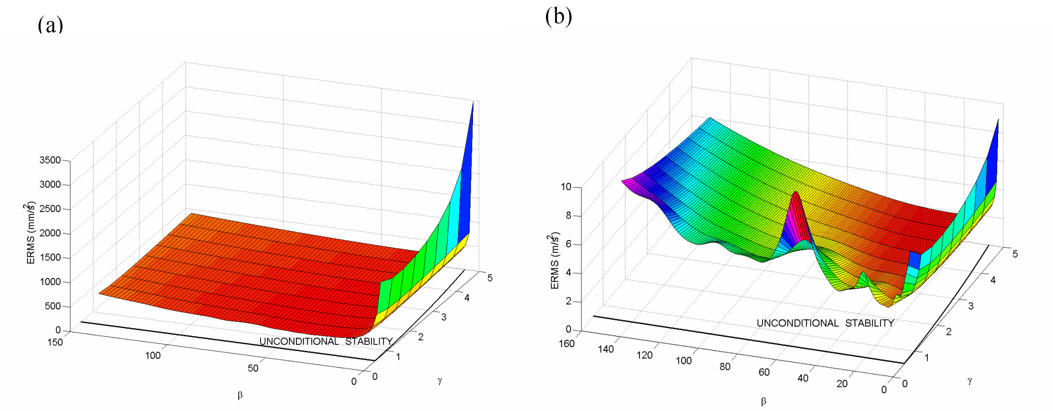


Fig. 5. RMSE evolution with γ and β parameters. (a) Signal 1. (b) Signal 2.

Future works will need to focus on the possibilities of producing an automatic algorithm, and on the embedding the algorithm in commercial biomechanical analysis packages. Namely, techniques will be devised to automatically choose the γ and β parameters for smoothing and differentiation of the given signal. The frequency transfer function of the Newmark method must be obtained, if possible, to automatically choose the value of γ and β parameters so as to obtain a convenient cut-off frequency for a given displacement signal. In conclusion, we believe that the biomechanics community will benefit from this new application of the Newmark integration method.

References

[1] Hatze, H., The fundamental problem of myoskeletal inverse dynamics and its implications, Journal of Biomechanics, 35, 2002, 109–115.

[2] Vaughan, C.L., Smoothing and differentiation of displacement-time data: an application of splines and digital filtering, International Journal of Bio-Medical Computing, 13, 1982, 375–386.

[3] D'Amico, M., Ferrigno, G., Comparison between the more recent techniques for smoothing and derivative assesment in biomechanics, Medical and Biological Engineering and Computing, 30, 1992, 193–204.

[4] Giakas, G., Baltzopoulos, V., A comparison of automatic filtering techniques applied to biomechanical walking data, Journal of Biomechanics, 30, 1997, 847–850.

[5] Giakas, G., Baltzopoulos, V., Improved extrapolation techniques in recursive digital filtering: a comparison of least squares and prediction. Journal of Biomechanics, 31, 1998, 87–91.

[6] Adham, R.I., Shihab, S.A., Discrete wavelet transform: a tool insmoothing kinematic data, Journal of Biomechanics, 32, 1999, 317–321.

[7] Alonso, F.J., Del Castillo, J.M., Pintado, P., Application of singular spectrum analysis to the smoothing of raw kinematic signals, Journal of Biomechanics, 38, 2005, 1085–1092.

[8] Newmark, N.M., A method of computation for structural dynamics, J. Engng. Mech. Div., Proc. ASCE, 85(EM3), 1959, 67–94.

[9] Géradin, M., Rixen, D., Mechanical vibrations, Wiley, Paris, 1994.

Francisco Javier Alonso, José María Del Castillo

Departamento de Ingeniería Mecánica, Energética y de los Materiales.

Universidad de Extremadura, Avda. de Elvas s/n, 06071 Badajoz, Spain

fjas@unex.es , delcasti@unex.es

Javier Cuadrado

Departamento de Ingeniería Industrial II

Universidad de La Coruña, Mendizábal s/n, 15403 Ferrol, Spain

javicuad@cdf.udc.es