# Efficiency Improvement in Flexible Multibody Dynamics by Means of Shape Integrals Preprocessing

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**Abstract**
The addition of flexible bodies to the simulation of multibody systems can affect seriously the CPU-time. The present work focuses on the introduction of a substantial improvement in two existing methods, one based on natural coordinates and another based on relative coordinates, that use finite element models –to which modal reduction is applied– attached to a floating reference frame. By means of shape integrals preprocessing, constant terms from the mass matrix and the velocity dependent inertia forces vector can be extracted, which makes the number of operations needed at each time-step for their computation dependent only on the number of modes, and not on the finite element mesh size. Both methods have been tested by simulating a vehicle with up to 12 flexible elements, observing that efficiency can be considerably increased, especially in the case of large finite element models.

**Keywords:** Flexible Multibody Dynamics, Floating Frame of Reference, Efficiency.

## 1. Introduction

The main purpose of this work is to introduce and evaluate an efficient method for the calculation of the mass matrix, $\mathbf{M}$, and the velocity dependent inertia forces vector, $\mathbf{Q}_v$, in the simulation of flexible multibody systems. In previous works [1,2], the authors developed two formulations based on the Floating Frame of Reference (FFR) approach [3], one in absolute (natural) coordinates, and another in relative coordinates. In both methods, component mode synthesis (Craig-Bampton reduction [4]) is used to reduce the number of coordinates. The problem is that $\mathbf{M}$ and $\mathbf{Q}_v$ are obtained at each time-step by means of a projection of the finite element (FE) nodal velocities into the vector of the body coordinates $\mathbf{q}$, operation which requires the evaluation of products involving matrices whose size is that of the FE model, thus reducing the efficiency because the modal reduction is not fully taken advantage of.

In this work, the implementation of shape integrals preprocessing [3] for the calculation of $\mathbf{M}$ and $\mathbf{Q}_v$, in both the absolute and the relative formulations, is described and discussed. In addition, a comparison in terms of efficiency is carried out between both formulations, with and without preprocessing.

## 2. Description of the Formulations

The equations of motion, according to an index-3 augmented Lagrangian formulation, are stated in the form [5],

$$\mathbf{M}\mathbf{q}+\boldsymbol{\Phi}_{\mathbf{q}}^{T}\alpha\boldsymbol{\Phi}+\boldsymbol{\Phi}_{\mathbf{q}}^{T}\boldsymbol{\lambda}^{*}=\mathbf{Q} \tag{1}$$

where $\mathbf{q}$ is the coordinates vector, $\mathbf{M}$ is the mass matrix, $\boldsymbol{\Phi}$ is the closed-loop constraints vector, $\boldsymbol{\Phi}_{\mathbf{q}}$ is its Jacobian matrix, $\mathbf{Q}$ is the vector of elastic, externally applied and velocity dependent forces, and $\boldsymbol{\lambda}^{*}$ is the Lagrange multipliers vector, obtained from an iteration process carried out within each time-step,

$$\boldsymbol{\lambda}_{i+1}^{*}=\boldsymbol{\lambda}_{i}^{*}+\alpha\boldsymbol{\Phi}_{i+1} \qquad i=0,1,2,... \tag{2}$$

which starts with $\boldsymbol{\lambda}_{0}^{*}$ equal to the value of $\boldsymbol{\lambda}^{*}$obtained in the previous time-step.

Both the absolute and relative formulations share the same FFR approach for flexible body modeling. The absolute position of any given point $\mathbf{r}$ of a flexible body can be obtained as,

$$\mathbf{r}=\mathbf{r}_{0}+\mathbf{A}\overline{\mathbf{r}}=\mathbf{r}_{0}+\mathbf{A}\left(\overline{\mathbf{r}}_{u}+\delta\overline{\mathbf{r}}\right) \tag{3}$$

where $\mathbf{r}_0$ stands for the position of the origin of the local frame of reference, $\mathbf{A}$ is a rotation matrix defined by the three orthogonal unit vectors of the frame [**u**|**v**|**w**], $\overline{\mathbf{r}}_{u}$ is the undeformed position of the point in local coordinates, and $\delta\overline{\mathbf{r}}$ is its local elastic displacement. The elastic displacement at a point is approximated by using component mode synthesis, which can be written in matrix form,

$$\delta\overline{\mathbf{r}}=\mathbf{X}\mathbf{y} \tag{4}$$

being $\mathbf{X}$ a matrix whose columns are the mode shapes, and $\mathbf{y}$ the vector of modal amplitudes.

The position and deformation status of the body can be described by a vector $\mathbf{q}$,

$$\mathbf{q} = \begin{Bmatrix} \mathbf{q}_r \\ \mathbf{y} \end{Bmatrix} \tag{5}$$

containing the rigid body variables $\mathbf{q}_r$, which depend on the formulation (absolute or relative) and define the position and orientation of the frame of reference, and the modal amplitudes $\mathbf{y}$, the same in both formulations.

A linear relationship, different for each formulation, can be established between the velocity of any point of the body $\dot{\mathbf{r}}$, and the velocities $\dot{\mathbf{q}}$,

$$\dot{\mathbf{r}} = \mathbf{B}(\mathbf{q})\dot{\mathbf{q}} \tag{6}$$

This relationship can be substituted into the kinetic energy expression, to obtain the mass matrix $\mathbf{M}$,

$$T = \frac{1}{2}\int_V \dot{\mathbf{r}}^T \dot{\mathbf{r}}\; dm = \frac{1}{2}\int_V \dot{\mathbf{q}}^T \mathbf{B}^T \mathbf{B} \dot{\mathbf{q}}\; dm \quad \Rightarrow \quad \mathbf{M} = \int_V \mathbf{B}^T \mathbf{B}\; dm \tag{7}$$

If the Lagrange equations are applied to the kinetic energy, the velocity dependent inertia forces can also be obtained,

$$\mathbf{Q}_v = -\int_V \mathbf{B}^T \dot{\mathbf{B}} \dot{\mathbf{q}}\; dm \tag{8}$$

These integrals are calculated by applying the co-rotational approximation proposed by Géradin and Cardona [6]. According to this approximation, the finite element interpolation matrices, intended for displacements, can be used also to interpolate velocities among neighbor nodes. This approximation becomes more exact as the FE mesh is refined, and is reasonably good for practical use. Assuming this, the **B** matrix integrals can be performed using the FE interpolation functions.

### 2.1. B Matrix Method

The approach used in [1,3] calculates **B** at every time-step, at all nodes, and assembles it in a full $\mathbf{B}_f$ matrix. Since the FE constant mass matrix $\mathbf{M}_f$ is the integral of the mass within all the body volume $V$, the following expressions for the mass matrix and the inertia forces can be obtained,

$$\mathbf{M} = \mathbf{B}_f^T \mathbf{M}_f \mathbf{B}_f \quad ; \quad \mathbf{Q}_v = -\mathbf{B}_f^T \mathbf{M}_f \dot{\mathbf{B}}_f \dot{\mathbf{q}} \tag{9}$$

This method is very easy to implement, but has an important drawback: these operations must be performed at each integrator step, and since the size of $\mathbf{B}_f$ and $\mathbf{M}_f$ is the same as the number of degrees of freedom of the FE model, this method partially eliminates the improvement introduced by the modal reduction.

### 2.2. Preprocessing Method

The approach proposed in this work uses a preprocessing stage to reduce the number of operations. After developing Equations (7) and (8), the body variables $\mathbf{q}$ and $\dot{\mathbf{q}}$ can be taken out from the integrals, since they are constant for all the points of the body. All the variable terms of the mass matrix and the forces vector can be calculated from ten constant shape integrals: the integral of the modes $\mathbf{\Gamma}$, three integrals of the undeformed position times the modes $\mathbf{\Gamma}_{\bar{x}}, \mathbf{\Gamma}_{\bar{y}}, \mathbf{\Gamma}_{\bar{z}}$, and six integrals of products between the directions of the modes $\mathbf{\Gamma}_{xx}, \mathbf{\Gamma}_{xy}, \mathbf{\Gamma}_{xz}, \mathbf{\Gamma}_{yy}, \mathbf{\Gamma}_{yz}, \mathbf{\Gamma}_{zz}$. The form of the integrals is as follows,

$$\mathbf{\Gamma} = \int_V \mathbf{X}\, dm \quad ; \quad \mathbf{\Gamma}_{\bar{x}} = \int_V \bar{x}_u \mathbf{X}\, dm \quad ; \quad \mathbf{\Gamma}_{xy} = \int_V \mathbf{X}_x^T \mathbf{X}_y\, dm \tag{10}$$

For example, the integral of the deformed local position, which appears in the mass matrix of the global formulation, is,

$$\int_V \bar{\mathbf{r}} dm = \int_V \left( \bar{\mathbf{r}}_u + \mathbf{X}\mathbf{y} \right) dm = m\bar{\mathbf{r}}_u^G + \mathbf{\Gamma}\mathbf{y} = m\bar{\mathbf{r}}^G \tag{11}$$

In this expression, $\bar{\mathbf{r}}_u$ is the undeformed local position, $m$ is the mass of the solid, and $\bar{\mathbf{r}}_u^G$, $\bar{\mathbf{r}}^G$ are the local positions of the center of mass in the undeformed and deformed configurations respectively.

All the remaining terms appearing in the mass matrix and the inertia forces vector can be calculated in a similar way, performing operations with matrices of the size of $\mathbf{y}$, being much less time consuming than the full $\mathbf{B}_f$ matrix projection.

## 3. Test Example and Results

The Iltis vehicle [7], which is a well known benchmark model for multibody system dynamics, is used as the base system for the tests. The vehicle consists of four identical suspensions, having each of them three flexible elements: the A-arm, the steering rod (fixed in the rear suspensions), and the upper bar which links the top of the hub to the chassis. All the elastic bodies have been modeled using 3D beam elements, with $n_e$ elements per bar. The steering rods and upper links are modeled as beams, with two static transversal modes and the first four dynamic ones. The A-arm consists of two bars, coincident at the hub connection, with one added element for the damper attachment ($2n_e$+1elements), and it has two vertical static modes (damper and hub attachments) and two dynamic ones. This makes a total of 12 flexible bodies, with $16n_e+4$ finite elements and 64 modes.

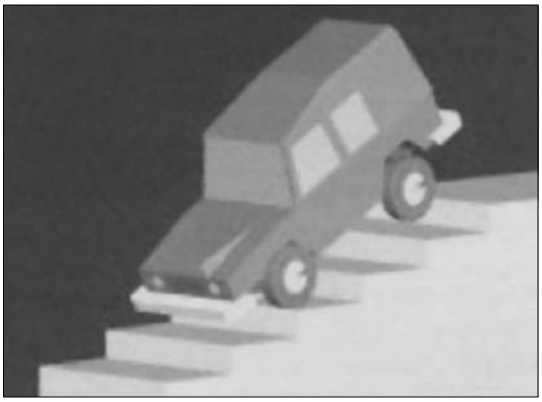

***Figure 1.*** *Iltis vehicle.*

In the test, the Iltis runs over the road profile shown in Figure 2, with an initial velocity of 5 m/s. Simulation is carried out by using both the absolute and the relative formulations, with and without preprocessing, with a time-step of 10 ms.

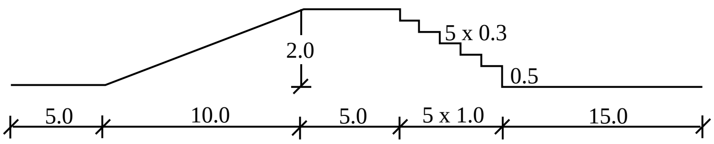


***Figure 2.*** *Road profile for the Iltis simulation.*

The results obtained from the simulations are the same, regardless of the use of preprocessing. The trajectories of the chassis origin and the center of the front left wheel obtained in both cases have been compared, with no variation between them. This is because the operations performed for obtaining the mass matrix and the inertia forces are exactly the same, being the only difference that the preprocessing eliminates repeated operations by calculating them before the execution time.

As it can be seen in Table 1 and Figure 3, the preprocessing makes the CPU-time invariant with respect to the number of elements. Both the absolute and the relative formulations benefit from this improvement, especially in the case of large finite element models, where the **B** matrix projection takes a significantly larger amount of time.

*Table 1.* *CPU-times for different finite element mesh sizes.*

| Elements ($n_e$) | 5 | 10 | 50 | 100 |
|---|---|---|---|---|
| Absolute, B | 5.34 | 6.59 | 36.78 | 131.59 |
| Absolute, P | 4.69 | 4.70 | 4.65 | 4.63 |
| Relative, B | 1.52 | 1.77 | 12.05 | 64.94 |
| Relative, P | 1.11 | 0.99 | 0.98 | 0.99 |

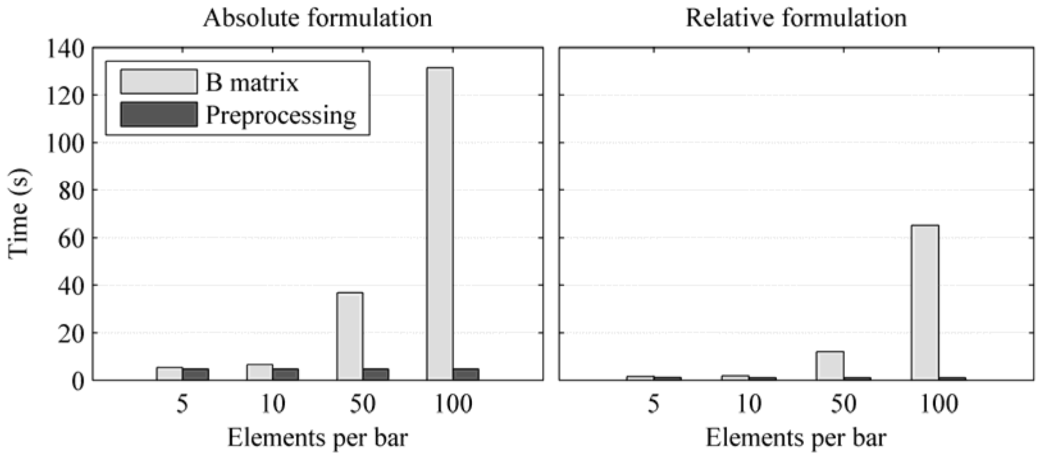


***Figure 3.*** *CPU-time vs number of finite elements.*

## 4. Conclusions

From the results obtained, it can be said that the preprocessing improves very significantly the performance in all cases. When very large finite element models are used, the **B** matrix method can become unpractical, whereas the preprocessing keeps the CPU-time dependent only on the number of modes.

The preprocessing times, which could be the only drawback of the proposed method, are negligible, if they are compared to the calculation of the mode shapes by solving the finite element system, since all the integrals can be obtained by direct matrix multiplication. In the present work, preprocessing has been done in MATLAB, and it takes less than 0.02 s for an A-arm with 100 elements per bar (i.e. 201 elements).

## 5. References


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