EXTENDED-KALMAN-FILTER OBSERVERS FOR MULTIBODY DYNAMICAL SYSTEMS

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Abstract

This work addresses the state estimation of multibody mechanical systems. These systems are described by second-order Lagrangian equations in dependent, constrained coordinates. The proposed observer, based on the extended Kalman filter (EKF), is described by first-order differential equations, with independent, non-constrained coordinates.

These different kinds of models pose the main problem when designing the observer. The approach followed is based on intimately relate the EKF realization with efficient methods for dynamic formulation and fast implementation of multibody systems: the velocity projection and the penalty methods. Simulation tests show robustness of the observer and promising ability for handling complex mechanical systems.

1 Introduction

The extended Kalman Filter (EKF) has been widely used, in combination with nonlinear dynamic models of systems, as state observer in several fields. The EKF for a nonlinear plant is in fact a simple linear Kalman Filter (KF) applied to the linearization of the plant around the trajectory given by the estimated state.

So, the main advantage of the EKF is that it inherits the optimality of the KF (optimality against state and measurement noise). Due to the linearization feature, this optimality becomes only local, not global. This implies the appearance of certain (usually large, but finite) “Domain of Attraction (DOA)”, that is, certain region of guaranteed convergence, which can be numerically precomputed (Delgado and Barreiro, 2003). In this way, the EKF provides robust observers for nonlinear systems.

In current practice, the EKF is combined with simplified dynamic models of the systems and elementary numerical integration schemes in order to streamline convergence and to achieve real-time performance of the computation process.

However, current state-of-the-art knowledge in Multibody System Dynamics opens the possibility of considering complex multibody models in real-time state observer applications, as long as specialized schemes (that combine dynamic formulations and numerical integrators to produce robust and efficient algorithms) are employed (Cuadrado, Cardenal, Bayo, 1997). The advantage of using such complex multibody techniques is that more information can be extracted from the model.

The EKF is typically formulated for first order nonlinear systems and non-constrained coordinates, in state-space form (ordinary differential equations, ODEs). Of course, the equations of a multibody system (differential algebraic equations, DAEs) can always be expressed in state-space form (minimum number of coordinates), and the corresponding second order equations can be converted into a first order one, duplicating the number of variables.

However, practicability of the proposed strategy is limited by the ability of the resulting formalism to provide fast numerical execution and real-time performance. For example, reduction of the multibody DAE to a state-space ODE as in (Haug, Negrut, Iancu, 1997) implies increasing the level of nonlinearity, which may be a serious drawback for complex multibody models and might affect the observer DOA size and convergence.

In the applications reported in the literature, the combination of the EKF with constrained DAE plants is usually addressed from the EKF point of view. That means adapting the KF rationale to the specific DAE problem. For example, (Nikoukhah, Willsky, Levy, 1990) show that the descriptor dynamics give rise to singular measurement noise covariance, and an extended maximum-likelihood method is applied. This same idea is followed in (Chiang et.al., 2002) where the constraint (unit quaternion norm) is treated as a pseudomeasurement. In (De Geeter, Van Brussel, De Schutter, 1997) the error from constraint linearization is treated in a separate step, after the EKF, increasing the computational complexity.

In this work, the solution to the combination of EKF and DAEs is approached from the DAE point of view. This is an advantage for complex multibody systems. As any observer runs in real-time a copy of the plant, the same techniques that are useful for modelling and fast simulation of complex multibody systems will also be useful for implementing observers for such systems.

So, this is the main idea of our approach, based on intimately relate the EKF realization with efficient methods for dynamic formulation and fast execution of multibody systems. In particular, this work reports the EKF formal derivation in the case of the velocity projection method and in the case of the penalty method.

Although the final objective of our project is addressing complex multibody systems in automotive applications, in this first preliminary work, for clarity, a simple example is considered. This test example is a four-bar mechanism with a spring-damper element, so that conclusions based on this simple system can later serve to address larger and more complex systems.
Two computational versions of the mechanism are created: the first one represents the real "prototype", while the second one plays the role of the "model". As the EKF provides robust observation, it is not necessary an exact identity between model and prototype. The physical parameters and exogenous forces may be different up to some extent. The objective is that the model follows the motion of the prototype with the help of an EKF. Preliminary numerical results and practical discussions are presented at the end of the paper.

2 EKF observer and multibody dynamics

Consider the system (plant) given by:

$$\begin{aligned}\dot{x}(t) &= A(t)x(t) + B(t)u(t) + \delta(t)\\ y(t) &= C(t)x(t) + \varepsilon(t)\end{aligned} \qquad (2.1)$$

Where $x(t)$ is the (unknown) state vector, and $u(t),y(t)$ are the known input and measurement variables. The matrix coefficients $A(t),B(t),C(t)$ are also known and the equations are affected by state and measurement noises $\delta(t),\varepsilon(t)$ with zero mean and given covariances Θ,Ξ, respectively. Then, the classical *linear* Kalman filter (KF) observer is given by (Bryson and Ho, 1975):

$$\begin{gathered}\dot{\hat{x}}(t) = A(t)\hat{x}(t) + B(t)u(t) + K(t)\left(y(t) - C(t)\hat{x}(t)\right)\\ K(t) = P(t)C^T(t)\Xi^{-1}(t)\\ \dot{P}(t) = A(t)P(t) + P(t)A^T(t) - P(t)C^T(t)\Xi^{-1}(t)C(t)P(t) + \Theta(t)\end{gathered} \qquad (2.2)$$

Notice that the state estimation has a prediction-correction structure, where the prediction is a copy of the plant *(Ax+Bu)* and the correction depends on the output error, affected by the Kalman gain $K(t)$. The main feature of the KF is its optimality: it minimizes the covariance $P(t)$ of the state-estimation error. When the plant is nonlinear:

$$\begin{aligned}\dot{x}(t) &= f\left(x(t),u(t)\right) + \delta(t)\\ y(t) &= h\left(x(t)\right) + \varepsilon(t)\end{aligned} \qquad (2.3)$$

Then, the *extended* Kalman filter (EKF) is given by (Bryson and Ho, 1975):

$$\begin{gathered}\dot{\hat{x}}(t) = f\left(\hat{x}(t),u(t)\right) + K(t)\left(y(t) - h\left(\hat{x}(t)\right)\right)\\ K(t) = P(t)C^T(t)\Xi^{-1}(t)\\ \dot{P}(t) = A(t)P(t) + P(t)A^T(t) - P(t)C^T(t)\Xi^{-1}(t)C(t)P(t) + \Theta\end{gathered} \qquad (2.4)$$

But now the matrices $A(t),C(t)$ are computed as the Jacobians of f, h with respect to the state, and are evaluated at the estimated trajectory (the true trajectory is not known).

The justification of the EKF is based on the principle of linearization. Actually, the EKF is nothing more that a linear KF applied to the precise linear plant (2.1) obtained linearizing the true plant (2.3) around the estimated trajectory.

If the state estimation error is small, then the linearization error is small as well, and KF and EKF are actually the same. For this reason, it can be said that the EKF is *locally optimal*, or optimal for small errors.
But if for some reason (level of noise, disturbance, etc.), the estimation error becomes large, it might happen that the linearization error is so large that optimality is deteriorated and even stability is lost. If fact, EKF observers present, due to nonlinearity, a more or less large Domain of Attraction (DOA) that can be precomputed or estimated (Delgado and Barreiro, 2003).

Some particular arrangements could be applied for improving convergence and robustness, as for example including the *forgetting factor* term $\lambda(t)P(t)$ in the last equation in (2.4). Strong nonlinearity or problems with observability should be addressed by more specific techniques (like sliding-mode observers, bounded-gain forgetting, etc.).

Now, the Multibody mechanical systems considered here are described by the constrained Lagrangian equations:

$$\begin{aligned} &M\ddot{q} + \Phi_q^T(q)\lambda = Q(q,\dot{q},u) \\ &\Phi(q) = 0 \end{aligned} \tag{2.5}$$

where $q, \dot{q}, \ddot{q}$ are positions, velocities and accelerations of a multibody system subject to the constraints $\Phi(q)=0$, so that λ denote the Lagrange multipliers. The positive semidefinite matrix M is the constant mass matrix and the right-hand side $Q(.)$ denote the generalized forces and momenta, including external known forces encoded in $u(t)$. The second order system of equations can be written as a first order one, putting $x^T=(q^T, v^T=dq^T/dt)$ and

$$\begin{pmatrix} \dot{q} \\ M\dot{v} \end{pmatrix} + \begin{pmatrix} 0 \\ \Phi_q^T(q) \end{pmatrix} \lambda = \begin{pmatrix} v \\ Q(q,\dot{q},u) \end{pmatrix} \tag{2.6}$$
$$\Phi_q(q)v = 0, \quad \Phi(q) = 0.$$

Both formulations (2.5) and (2.6) are obviously equivalent. The integral constraint $\Phi(q)=0$ has been complemented with the differential one $d\Phi/dt=\Phi_q(q)v=0$. In this way, counting pairs of positions and velocities, the total number of dependent states $2n_d$ is subject to $2n_c$ constraints so that there are $2n_i$ free independent states.

Once the second order equations have been converted into first order, the main problem to match (2.6) (plus some noise signals) to (2.3), and derive the EKF, is the presence of constraints and the corresponding Lagrange multiplier term in (2.6). This problem can be dealt with in several ways.

Of course, one can always eliminate the constraints and the Lagrange multipliers in (2.6). However this is analytically involved (Haug, Negrut, Iancu, 1997) and in practice it increases the degree of nonlinearity which might affect the observer DOA size, and reduce convergence.

One may resort as well to specific solutions for Kalman filtering of Differential-Algebraic Equations (DAEs), as in the literature references discussed in the introduction, however the explicit treatment of the model as a DAE would increase conceptual and computational complexity.
Our approach is motivated by the availability of new robust and efficient algorithms for dynamic formulation and numerical integration of multibody systems (Cuadrado, Cardenal, Bayo, 1997). And the main reason for choosing this approach is very simple to state: *As any observer runs in real-time a copy of the plant, then the techniques that are useful for efficient simulation of complex multibody systems will be useful as well for implementing observers for the same systems.*

Following this idea, the sections 3 and 4 report the formal derivation of EKF observers for Multibody systems in the case of the velocity projection (R-matrix) method and in the case of the Penalty method.

3 EKF observer based on the r-matrix

The main idea in this method, known as the 'velocity proyection method' or the 'R-matrix method' (García de Jalón and Bayo, 1994) is to obtain an ODE with dimension n_i equal to the actual number of degrees of freedom (DOF), using a set z of independent coordinates. The starting point is to establish the following relation between velocities:

$$\dot{q} = R\dot{z} \tag{3.1}$$

Where $q(t)$ are all the n_d dependent variables and $z(t)$ is a set of n_i independent variables. Such relation (3.1) can always be found, for instance, taking the derivative of the restrictions, $0=\Phi_q(q)\dot{q}$, and splitting all the velocities in two subsets, such that one subset of velocities can be written as a function of the other subset. Once (3.1) is obtained, it follows that

$$\ddot{q} = \dot{R}\dot{z} + R\ddot{z}$$

And, going back to (2.5)

$$MR\ddot{z} + M\dot{R}\dot{z} + \Phi_q^T\lambda = Q$$

Premultiplying by the transpose of R and having in mind that $\Phi_q R=0$, one reaches to

$$\ddot{z} = \left(R^T M R\right)^{-1}\left[R^T\left(Q - M\dot{R}\dot{z}\right)\right] = \bar{M}^{-1}\bar{Q} \tag{3.2}$$

which defines implicitly the corrected mass matrix $\bar{M}$ and the corrected vector of generalized forces $\bar{Q}$.

So, the result is that the DAE (2.5) in the dependent variables has been converted into the ODE (3.2) expressed in independent variables. Then, the EKF in (2.4) can be straightforwardly applied. In particular, the state-space matrix is obtained as the linearization (evaluated at the estimated trajectory):

$$A = \begin{bmatrix} O & I \\ \dfrac{\partial\left(\bar{M}^{-1}\bar{Q}\right)}{\partial z} & \dfrac{\partial\left(\bar{M}^{-1}\bar{Q}\right)}{\partial \dot{z}} \end{bmatrix} \tag{3.3}$$

The detailed expressions of the previous partial derivatives are omitted for brevity. They are long complex expressions, but otherwise conceptually simple, as they can be computed by laborious derivation and application of the chain rule.

The main advantage of the R-matrix method is the reduction of the number of equations, at the expense of having to compute, at each instant, $R(t)$ and the dependent coordinates as functions of the independent ones. It also requires the effort of managing the redundancy in restrictions and the changes in the representative set of velocities.

4 EKF observer based on penalty method

The basic idea in the penalty method is to postulate that the constraining forces in (2.5) are proportional to the violation of the restrictions. In particular, the Lagrange multipliers are chosen in the form (García de Jalón and Bayo, 1994):

$$\lambda = \alpha\left(\ddot{\Phi} + 2\varsigma\omega\dot{\Phi} + \omega^2\Phi\right) \tag{4.1}$$

where α is the penalty factor, and it is usually fixed to a very large value, 10^6 or 10^7. Notice that the combination of the constraint function and their derivatives takes the form of a second order oscillating system with damping coefficient and natural frequency usually chosen as ζ=1,ω=10. So, the rigid constraints in the DAE (2.5) can be converted into non-rigid constraints in an ODE:

$$\ddot{q} = \left(M + \Phi_q{}^T \alpha \Phi_q\right)^{-1}\left[Q - \Phi_q{}^T \alpha\left(\dot{\Phi}_q \dot{q} + 2\varsigma\omega\dot{\Phi} + \omega^2\Phi\right)\right] \tag{4.2}$$

However, due to the very large value of α, it can be shown that this is equivalent to representing the constraints by springs of large stiffness and dampers of large friction coefficient. In this way, the constraints can actually be violated, but only in a very small amount, enough for representing de DAE (2.5) as the ODE (4.2) with negligible approximation errors.

Compared to the R-matrix method, the equation (4.2) has the drawback that the number of variables is larger: it is equal to the total number of dependent variables. However, this method has the advantage that (4.2) can be directly integrated as an ODE and it is not necessary to solve at each time instant the problems of passing from independent to dependent coordinates and related problems mentioned in the previous section. Furthermore, the corrected mass matrix (the inverted matrix in (4.2)) may be invertible, even if M is only positive semidefinite.

An EKF as (2.4) is straightforwardly derived for the ODE (4.2) using the standard procedure. In particular, the state-space matrix *A(t)* has the same form than (3.3) but replacing the corrected mass matrix and corrected generalized forces by the corresponding values derived from (4.2). For the sake of brevity, the details of the derivation of *A(t)* are omitted.

Notice that, in the covariance equation for *P(t)* in (2.4), the size of *P(t)* is now $n_d \times n_d$ so that the number of entries of *P(t)* might become now considerably larger than in the case of independent coordinates. As the size n_d of the problem increases, it affects particularly to *P(t),* with a number of entries proportional to the square of the size.

5 Example

A four-bar mechanism with a spring-damper element is chosen as example, see Fig.1. Two computational versions of the mechanism are created: the first one represents the real "prototype", while the second one plays the role of the "model". A sensor in the prototype provides as a measurement $y=s$, the displacement value of the spring-damper element, that is, the distance between point A and point 2.

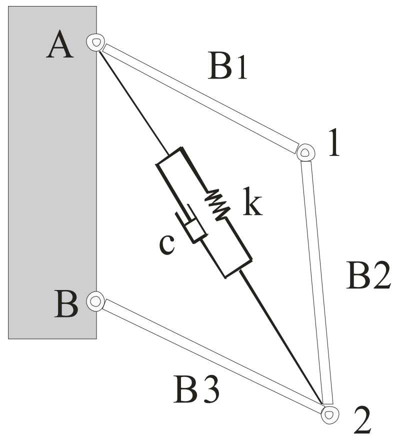

Figure 1. A four-bar mechanism with a spring-damper element.

The state variables in this example are the coordinates of points *1* and *2*, and the distance *s*:

$$q = \begin{pmatrix} x_1 & y_1 & x_2 & y_2 & s \end{pmatrix}^T$$

The inertial matrix of the whole mechanism is computed considering the contributions of the three 2D bars (García de Jalón and Bayo, 1994) and the zero contribution of the spring-damper element (of negligible mass):

$$M_1 = m_1\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 \\ 0 & 0 & 0 & 1/3 \end{pmatrix},$$

$$M_2 = m_2\begin{pmatrix} 1/3 & 0 & 1/6 & 0 \\ 0 & 1/3 & 0 & 1/6 \\ 1/6 & 0 & 1/3 & 0 \\ 0 & 1/6 & 0 & 1/3 \end{pmatrix}$$

$$M_3 = m_3\begin{pmatrix} 1/3 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Finally, the *5x5* global inertial mass matrix is:

$$M = \begin{pmatrix} \sum M_i & 0 \\ & \vdots \\ 0\cdots & 0 \end{pmatrix}$$

Notice that a fifth row and column of zeros has been added, as there is no inertia associated to the spring-damper element.

The generalized force vector is obtained considering the gravity as the only force that acts over each bar. So, for the three bars it results (García de Jalón and Bayo, 1994):

$$Q_1 = -\frac{1}{2}m_1 g\begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \quad Q_2 = -\frac{1}{2}m_2 g\begin{pmatrix}0\\1\\0\\1\end{pmatrix}, \quad Q_3 = -\frac{1}{2}m_3 g\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

On the other hand, the contribution of the spring-damper element is:

$$f = -k(s - s_0) - c\dot{s}$$

Where k is the elastic constant, c the damping coefficient and s_0 the natural spring length.

Then, the vector Q is:

$$Q = \begin{pmatrix} \sum Q_i \\ f \end{pmatrix}$$

Finally, the constraints and the constraint Jacobian are:

$$\Phi = \begin{pmatrix} (x_1 - x_A)^2 + (y_1 - y_A)^2 - L_1^2 \\ (x_2 - x_1)^2 + (y_2 - y_1)^2 - L_2^2 \\ (x_2 - x_B)^2 + (y_2 - y_B)^2 - L_3^2 \\ (x_2 - x_A)^2 + (y_2 - y_A)^2 - s^2 \end{pmatrix}$$

$$\Phi_q = \begin{pmatrix} 2(x_1 - x_A) & 2(y_1 - y_A) & 0 & 0 & 0 \\ 2(x_1 - x_2) & 2(y_1 - y_2) & 2(x_2 - x_1) & 2(y_2 - y_1) & 0 \\ 0 & 0 & 2(x_2 - y_B) & 2(y_2 - y_B) & 0 \\ 0 & 0 & 2(x_2 - x_A) & 2(y_2 - y_A) & -2s \end{pmatrix}$$

So, the Multibody mechanical system considered here is described by the equations (2.5) with the previous data.

The numerical values of the parameters are as follows. The masses are m_1=2 Kg, m_2=25 Kg, m_3=2 Kg, the bar lenghts are L_1=0.9 m, L_2=1.05 m, L_3=1m, the spring length is s_0=1.70m, the *(x,y)* fixed point coordinates are A = (0,0), B = (–1,0), the spring coefficient is k=10000 Nw/m and the damping coefficient value is c=500 Nw/(m/s).

Regarding the tuning of the EKF, the main parameters are the matrices Θ, Ξ. In principle, they have to represent the covariances of the zero-mean noises δ,ε in (2.1) or (2.3). However, in practice, this information is not always clearly known, so in fact the matrices Θ, Ξ. are used as tuning parameters adjusted by trial-and-error experimental work.

It should be stressed that there is no perfect solution to the filtering problem, but rather there exist trade-offs between competing objectives. Typically, the solutions that provide fast convergence (of the estimations to the true values) are affected by higher levels of noise. If low levels of noise are desired, then the initial errors converge more slowly to zero. To facilitate the tuning, typically the matrices are postulated to be diagonal, Θ=diag(θi), Ξ=diag(ξi).

After some trial-and-error test work under the simulation conditions to be detailed later, the EKF tuning is set to Θ=diag(*0.1*), Ξ=diag(*0.001*). An additive *forgetting factor* term $\lambda(t)P(t)$ has been added to the last equation in (2.4), with $\lambda=0.1$. The initial covariance value has been chosen in the form *P(0)=diag(diag(pi),diag(vi)),* to represent different initial uncertainties in positions and velocities. As it is supposed that model and prototype start from rest condition, the initial uncertainty in velocities is zero, and *0.1* in positions so that *P(0)=diag(diag(0.1),diag(0))*.

Regarding the simulation conditions, to show the recovering from different initial conditions, the real prototype starts at *s(0)=1.80m* and the virtual model (observer) starts at *s(0)=1.85m*. In the same way, to evaluate the effect of noise and error in measurements, it is supposed that $y=s+\varepsilon$, ,with measurement noise ε within the interval *[–0.02,+0.02]m (2cm*), uniformly distributed. Finally, to check what happens with uncertain exogenous forces, the prototype runs under normal gravity, *g=9.81m/s2*, but the observer runs under *g=8.81m/s2*. In these conditions, the time trajectory for three of the state variables (the real one and the estimated one) is plotted in Fig.2, Fig.3, Fig.4 (R-matrix method) and correspondingly in Fig.6, Fig.7, Fig.8 (Penalty method). Furthermore, to validate a more difficult situation, Fig.5 and Fig 9 show the evolution of *s(t)* and its estimation,, under a harder noise level in *[–0.1,+0.1]m* with the true or real coefficient is c=0 Nw/(m/s) (so that the prototype oscillates) but the model is taken c=100 Nw/(m/s).

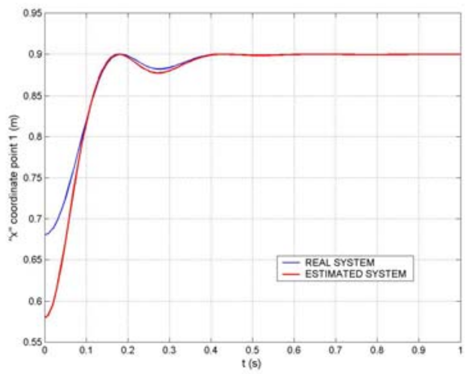

Figure 2. Evolution of x_1 variable in R-matrix method.

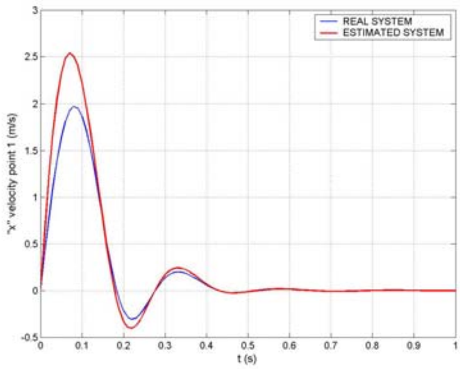

Figure 3. Evolution of x_1 velocity in R-matrix method.

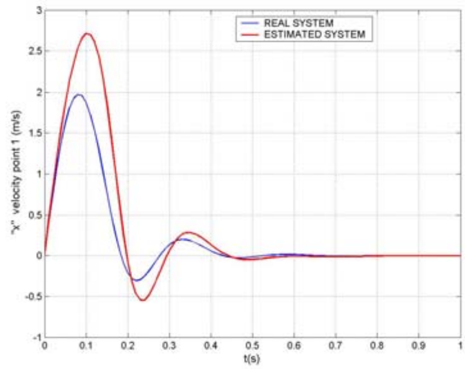

Figure 7. Evolution of x_1 velocity in penalty method.

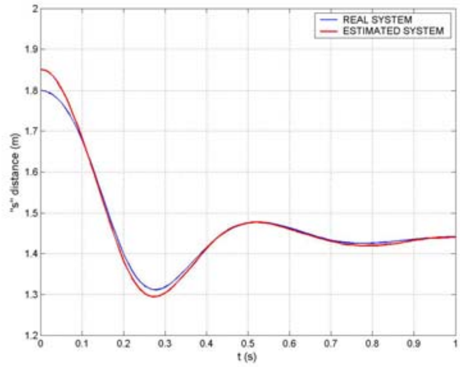

Figure 4. Evolution of *"s"* distance in R-matrix method.

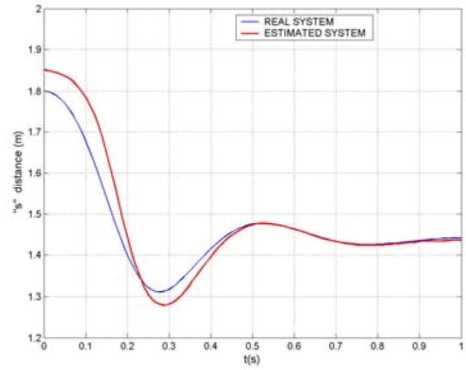

Figure 8. Evolution of *"s"* distance in penalty method.

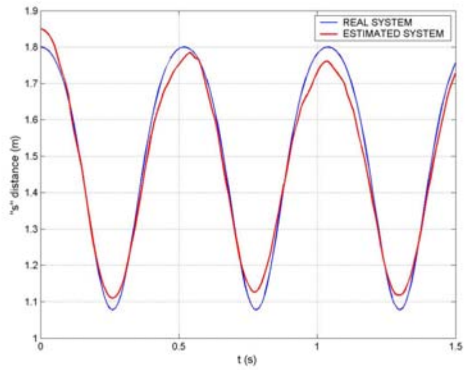

Figure 5. Evolution of *"s"* distance in R-matrix method (oscillating prototype)

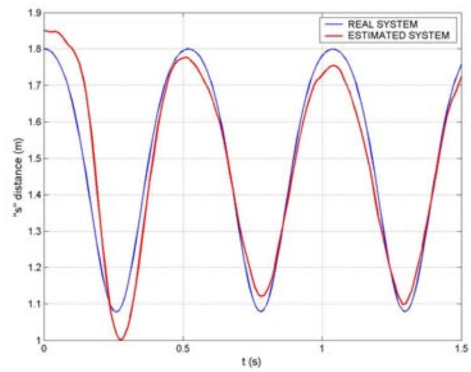

Figure 9. Evolution of *"s"* distance in penalty method (oscillating protoype).

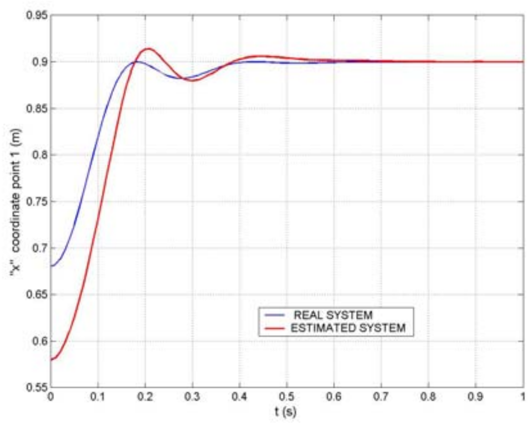

Figure 6. Evolution of x_1 variable in penalty method.

All the simulation plots show a reasonable good performance level, with satisfactory convergence speed, robustness against parameter uncertainty, and noise filtering and attenuation. The R-matrix method appears to be better than the Penalty method. It is believed that the reason in related to the relaxation of constraints in the Penalty method. This relaxation is in part traded off against estimation accuracy: If one desires good and fast estimation, the "correction forces" in the observer need to be large, and (although the penalizer is large, α=10e7) these forces produce a larger and different violation of constraints in the observer, compared to the prototype. This suggest to consider, for future work, dynamic formulations of Multibody systems that force the constraints to be satisfied strictly, like, for example in the Augmented Lagrangian method (García de Jalón and Bayo, 1994).

6 Conclusions

This work presents a study on the application of EKF observers to Multibody Dynamical Systems. Although the numerical tests have been carried out, by the moment, on a simple four-bar example, the final objective is to implement the observers on complex Multibody systems, such those arising in the field of automotive control.

So, having in mind complex Multibody dynamics, the approach has been based on the idea that the same techniques which are efficient for efficient simulation of such complex systems will be efficient as well for implementing the observers. Two methods have been chosen: the velocity projection or R-matrix technique and the penalty technique. The detailed development of the EKF observer for these methods has been presented.

The simulation tests show successful results in both cases. After a not very involved trial-and-error tuning, the EKF final observers are robust with respect to noise, initial estimation errors and even different input forces (*9.81m/s2* and *8.81m/s2* different gravity forces). The R-matrix method has the advantage of working with a lower number of variables and the Penalty method has the advantage that it does not require the passing from independent to dependent variables at each time sample.

The R-matrix method appears to be superior to the Penalty method and this suggests, from the discussion in the previous section, to consider in future work more efficient real-time Multibody formulations like, for example, the Augmented Lagrangian technique.

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