Techniques for the simulation of redundantly constrained multibody systems

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EXTENDED ABSTRACT

1 Introduction

In this work we deal with mechanical systems that are capable of rigid body motion, but they have more constraints than necessary to achieve the kinematic motion specifications. Such extra constraints are often associated with additional forces that enhance the structural integrity of the mechanical system at hand. These systems are called overconstrained or redundantly constrained. On the other hand, the issue of dealing with redundant constraints also comes into play even more frequently in the development of multibody system models using generic algorithms. For example, if we build the model of a "planar" four-bar linkage using a general three-dimensional rigid body algorithm then we arrive at a redundantly constrained model where some of the constraints are unnecessary from the kinematic point of view, as they are already enforced by other constraints.

Interestingly, the topic of redundant constraints and the algorithms that are able to handle such cases have received relatively little attention in the literature. An additional issue is that a redundantly constrained system may behave in some cases as a mechanism, i.e. capable of rigid body motion, and in other cases as a structure. In general, there are two possible ways to deal with redundant constraints. The first is to relax all or some of the constraints and directly represent the constraint forces with penalty systems, or to use the augmented Lagrangian formulation with artificial penalty masses and inertias. The second possibility is to determine a generalized resultant of all the constraint forces, where the components of the resultant correspond to the generalized constraint forces associated with the generalized velocities used to parameterize the system. This can be achieved via properly decomposing the generalized forces to parts associated with constrained and admissible motions of the system. In this paper, we discuss an algorithm that uses both of these concepts to achieve efficient simulation of mechanical systems.

2 Methodology

In this work, a single-step, semi-implicit forward Euler stepping scheme [1] is used to discretize the equations of motion with a time step h as:

$$\begin{bmatrix} \mathbf{I} & -h\mathbf{N} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} & -\mathbf{A}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{A} & \overline{\mathbf{C}}_{\Phi} \end{bmatrix} \begin{bmatrix} \mathbf{q}_{n+1} \\ \mathbf{v}_{n+1} \\ h\boldsymbol{\lambda}_{n+1} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_n \\ \mathbf{M}\mathbf{v}_n + h\left(\mathbf{f}_a - \mathbf{c}\right) \\ \overline{\mathbf{B}}_{\Phi}\boldsymbol{\Phi} \end{bmatrix} \tag{1}$$

where $\mathbf{I}$ is an identity matrix, $\mathbf{M}$ stands for the mass matrix, $\mathbf{v}$ is a vector containing a set of generalized velocities, $\boldsymbol{\Phi}$ is the vector of constraint equations, $\mathbf{A}$ is the Jacobian matrix, $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers, and $\mathbf{f}_a$ and $\mathbf{c}$ stand for the vectors of generalized applied forces and Coriolis and centrifugal effects. In the general case, the relation between the generalized coordinates of the system ($\mathbf{q}$) and the set of generalized velocities ($\mathbf{v}$) is described through velocity transformation, $\dot{\mathbf{q}} = \mathbf{N}\mathbf{v}$. Following a penalty approach, the Lagrange multipliers are assumed to be proportional to the violation of the constraints. Diagonal matrices $\overline{\mathbf{C}}_{\Phi}$ and $\overline{\mathbf{B}}_{\Phi}$ contain the compliance corresponding to each constraint and damping terms for the stabilization of the integration.

The use of the above discretization is motivated by the need of performing the simulation with a minimum computational effort, to enable real-time simulation and animation of the motion, as well as other features such as contact detection.

2.1 Test Problem

The described formulation was used for the dynamic simulation of a Bricard mechanism [2], a well-known, 1-degree of freedom overconstrained system. It is composed of five bars of length $l = 1$ m and mass $m = 1$ kg, connected by revolute joints. One of the constraint equations can be expressed as a linear combination of the others at any moment during motion.

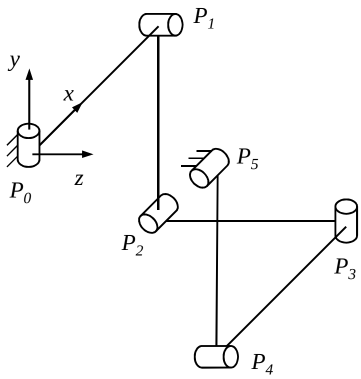

Figure 1: *Bricard's mechanism*

Although the Chebychev-Kutzbach-Grübler criterion predicts zero degrees of freedom for this system, the particular orientation of the joints allows for a smooth movement without singularities. Starting from the position depicted in Fig. 1 and under gravity effects, point P_2 theoretically describes a periodic movement between $x = 1$ m and $x = -1$ m. However, a 10 s simulation with $h = 10^{-2}$ s showed that the obtained violation of constraints of $\|\boldsymbol{\Phi}\| < 10^{-4}$ is enough to convert the system of equations of Eq. (1) into an incompatible one, turning the mechanism into a structure for certain regions of its theoretical motion range. Indeed, this also corresponds to the physical fact that the existence of imperfections in the geometry of the rigid joints can prevent the Bricard mechanism from moving.

2.2 Corrective Methods and Results

Two methods were developed and tested to overcome the problems stemming from the presence of redundant constraints: (a) the addition of flexibility to the joints and (b) the exact fulfilment of the constraint equations. First, the kinematic constraints were relaxed; it was found that the addition of angular compliance to the joints yielded the predicted behaviour of the mechanism. Alternatively, a second strategy, based on the projection of the positions and velocities obtained by the stepping scheme on the subspace of admissible motion (SAM) of the configuration space [3], was used. The integrated positions $\mathbf{q}_{n+1}$ given by Eq. (1) are projected onto the SAM by means of a fixed-point iteration process:

$$\mathbf{q}_{n+1}^{k+1} = \mathbf{q}_{n+1}^{k} - \mathbf{W}^{k}\mathbf{\Phi}^{k} \tag{2}$$

with matrix $\mathbf{W} = \mathbf{L}^{-1}\left(\mathbf{A}\mathbf{L}^{-1}\right)^{\dagger}$. Matrix $\mathbf{L}$ is the result of the Cholesky decomposition $\mathbf{M} = \mathbf{L}^{\mathrm{T}}\mathbf{L}$, and the symbol † stands for the pseudo-inverse of a matrix. Velocities are subsequently projected, making use of the projection matrix $\mathbf{P}_a = \mathbf{I} - \mathbf{W}\mathbf{A}$:

$$\mathbf{v}_{n+1}^{proj} = \mathbf{P}_a\mathbf{v}_{n+1} \tag{3}$$

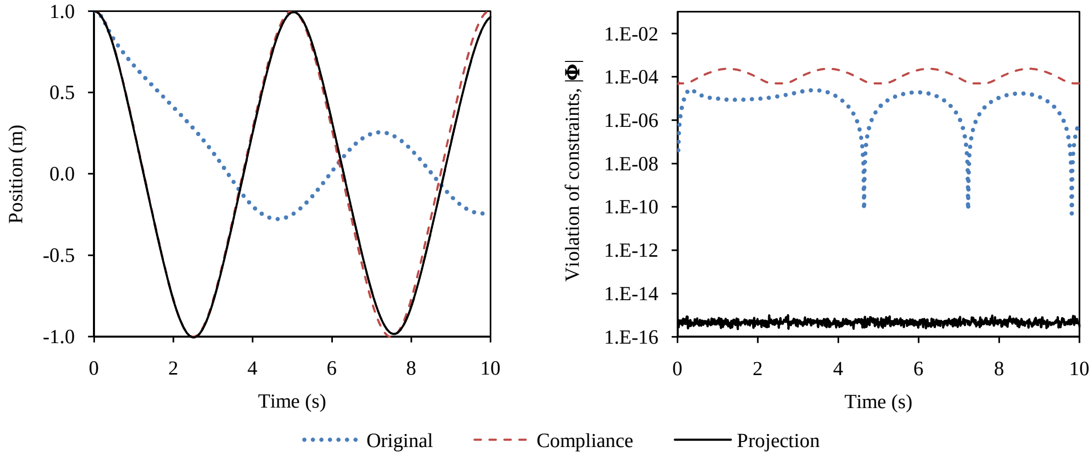

Figure 2: *Coordinate x of point P_2 and violation of constraints during the motion.*

The use of compliant joints led to the motion of the mechanism following its expected theoretical behaviour, although the violation of the constraints rises slightly with respect to the previous case. The projection of positions and velocities can reduce the violation of constraints to machine error, and ensures the right behaviour of the system without modifying the physical properties of the model, at the cost of higher computational effort. Results are summarized in Fig. 2.

References

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