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A complementarity formulation for rigid body contact problems and a solution algorithm

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The rigid body representation is extensively used to model and analyze machines and many other engineering systems. In mechanical engineering, a large and important class of problems involves the direct contact between two or more elements of the system at hand. The modelling of the contacts is of utmost importance. In rigid body systems such a contact can be represented with unilateral kinematic and force relations. The relative contact displacement can exist only in the normal direction at each contact point, and the normal contact force can only be compressive. In addition, the normal contact force and displacement also need to satisfy a complementarity condition, namely that only one of them can be different from zero at the same time. These facts combined with the first principles of mechanics lead to a mathematical model of the system which can be stated in the form of a complementarity problem to be defined below.

Given a function $T:\mathbb{R}^n \to \mathbb{R}^n$ and a closed convex cone $C \subset \mathbb{R}^n$, the complementarity problem consists of finding x such that

$$\begin{cases} x \in C \\ T(x) \in C^* \\ \langle T(x), x\rangle = 0, \end{cases}$$

where

$$C^* = \{z \in \mathbb{R}^n \mid \langle z, y\rangle \geq 0,\ \forall y \in C\}.$$

By a Linear Complementarity Problem (LCP) we understand that the function T is affine, meaning that $T(x) = Ax + b$ where A is a constant square matrix and b is a constant vector, and the closed convex cone C associated with the problem is the positive orthant

$$\mathbb{R}^n_+ = \{(x_1,\ldots,x_n) \in \mathbb{R}^n \mid x_i \geq 0 \quad \forall i\}.$$

The resulting complementarity problem in the model is relatively simple when no friction is considered at the contacts. This relies on the fact that in this case an LCP can be constructed representing the phenomenon truly. However, the complementarity problem when Coulomb friction is taken into account is more complex since the closed convex cone C might not be the positive orthant. It is common engineering experience that the Coulomb friction laws give realistic representation of friction in many systems (e.g. [1]). Detailed developments of complementarity problems for engineering systems are presented in [1], among others.

In the presence of friction, it is also common to derive an LCP. In this setting, simplex-like methods, originating from the algorithms of Lemke and Dantzig, are a well-known family of strategies for solving LCPs. As a matter of fact, finite polyhedral approximation of the friction cones leads to such LCPs (see, e.g., [2]). Nonetheless, these simplex-like methods might solve the problem in exponential time in the worst case, which is not desirable.

In order to overcome the above obstacle, a new formulation for the problem under consideration is developed in [3]. In this reference, the proposed model is based on a finite difference approximation of the equations of motion and at each discrete time a complementarity problem with a positive semi-definite matrix is solved, avoiding the polyhedral approximation of the friction cones. The solution of this complementarity problem, if exists, gives contact impulses and velocities. However, this approach works only if static friction occurs or the magnitude of the tangential velocity at contact is close enough to zero. The authors also propose a fixed-point solution technique to solve the resulting complementarity problem.

Following [3], in this work, we develop a mathematical model that can be truly representative of rigid body contact problems when Coulomb friction is used. This model is applicable in both static and dynamic cases. The resulting formulation is based on a complementarity problem whose function is affine and the closed convex cone associated with the problem is a combination of a positive orthant of a certain Euclidean space and friction cones. It is remarkable that the matrix A associated with the problem might not be positive semi-definite.

After introducing the model, we formulate the complementarity problem in terms of a certain variational inequality problem. Seeing the complementarity problem as a variational inequality problem leads us to find the solution of the complementarity problem with the iterative algorithm proposed in [4]. This algorithm combines line search and projection techniques to develop a solution strategy for the problem under consideration. The algorithm generates globally convergent sequences to a solution of the given problem under a set of standard assumptions. In particular, matrix A does not have to positive semi-definite.

The line search step in the solution algorithm is not expensive. However, the most difficult step of the solution algorithm at each iterate is the computation of the projection onto the set C, which is equivalent to solving a quadratic minimization problem. Fortunately, in our setting, this projection can be computed via a closed formula. Therefore, this step should be rather inexpensive in terms of the computational cost too.

The scheme was tested in the forward-dynamics simulation of examples involving both static and kinetic friction. Results showed that it is able to yield realistic results for systems with several contacts happening simultaneously.

References

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