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Manual and automatic direct-differentiation methods for the sensitivity analysis of multibody systems in independent coordinates

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Abstract

The efficiency of sensitivity analyses in the context of multibody systems is still a somewhat unexplored field. Compared to other areas like forward dynamics, inverse dynamics, control, or flexible dynamics, where very efficient formulations have been developed over the last decades, sensitivity analyses, for the most part, lack that kind of efficiency assessment. Two facts may serve as proof of this: on the one hand, the real-time computation of sensitivities is still unheard of; on the other hand, state-of-the-art multibody tools do not provide general-purpose sensitivity analysis modules yet. This paper aims at the development of a general-purpose sensitivity analysis method in natural coordinates via two different direct-differentiation techniques, namely manual differentiation and automatic differentiation. Together with the analysis of a real-life 18-DOF coach maneuver, useful insight is provided about the computational efficiency and accuracy of the formulations.

Equations of motion and direct-differentiation sensitivity equations

The equations of motion employed in this work were extensively described in [2]:

$$\bar{\mathbf{M}}(\mathbf{z},\mathbf{b})\ddot{\mathbf{z}} = \bar{\mathbf{Q}}(t,\mathbf{z},\dot{\mathbf{z}},\mathbf{b}) \tag{1a}$$

$$\bar{\mathbf{M}}(\mathbf{z},\mathbf{b}) = \mathbf{R}^{\mathrm{T}}\mathbf{M}\mathbf{R} \tag{1b}$$

$$\bar{\mathbf{Q}}(t,\mathbf{z},\dot{\mathbf{z}},\mathbf{b}) = \mathbf{R}^{\mathrm{T}}(\mathbf{Q}-\mathbf{M}\mathbf{S}\mathbf{c}) \tag{1c}$$

where $\mathbf{q}\in\mathbb{R}^n$ and $\mathbf{z}\in\mathbb{R}^f$ are the vectors of dependent and independent coordinates of the system respectively; $\mathbf{b}\in\mathbb{R}^p$ is the vector of design parameters; $\mathbf{R}\in\mathbb{R}^{n\times f}$ and $\mathbf{S}\in\mathbb{R}^{n\times m}$ are two matrices that can be calculated as explained in [2]; $\mathbf{M}(\mathbf{q})\in\mathbb{R}^{n\times n}$ and $\mathbf{Q}(t,\mathbf{q},\dot{\mathbf{q}},\mathbf{b})\in\mathbb{R}^n$ are the mass matrix and generalized forces vector in dependent coordinates; and $\mathbf{c}\equiv\mathbf{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} = -\dot{\mathbf{\Phi}}_{\mathbf{q}}\dot{\mathbf{q}}-\dot{\mathbf{\Phi}}_t$, where $\mathbf{\Phi}(t,\mathbf{q},\mathbf{b})\in\mathbb{R}^m$ is the vector of constraint equations that relates the dependent coordinates of the system and the subscript denotes partial differentiation.

The direct differentiation method (DDM) for the computation of *design sensitivities* was developed by Krishnaswami and Bhatti [5] and Chang and Nikravesh [1] in the mid-eighties. It is, in general, simpler to implement than the adjoint variable method (AVM). It consists of computing the vectors of *state sensitivities*, $\mathbf{q}_{\mathbf{b}}$, $\dot{\mathbf{q}}_{\mathbf{b}}$ and $\ddot{\mathbf{q}}_{\mathbf{b}}$, which then allow computing design sensitivities $\Psi_{\mathbf{b}}$, where Ψ is an objective function. Differentiating the equations of motion in Eq. (1) w.r.t. the design parameters and rearranging:

$$\bar{\mathbf{M}}\ddot{\mathbf{z}}_{\mathbf{b}} + \bar{\mathbf{C}}\dot{\mathbf{z}}_{\mathbf{b}} + \left(\bar{\mathbf{K}}+\bar{\mathbf{M}}_{\mathbf{z}}\ddot{\mathbf{z}}\right)\mathbf{z}_{\mathbf{b}} = \bar{\mathbf{Q}}_{\mathbf{b}} - \bar{\mathbf{M}}_{\mathbf{b}}\ddot{\mathbf{z}} \tag{2a}$$

$$\mathbf{z}_{\mathbf{b}}(t_0) = \mathbf{z}_{\mathbf{b}0} \tag{2b}$$

$$\dot{\mathbf{z}}_{\mathbf{b}}(t_0) = \dot{\mathbf{z}}_{\mathbf{b}0} \tag{2c}$$

where $\bar{\mathbf{K}}\equiv-\partial\bar{\mathbf{Q}}/\partial\mathbf{z}$, $\bar{\mathbf{C}}\equiv-\partial\bar{\mathbf{Q}}/\partial\dot{\mathbf{z}}$ and $\bar{\mathbf{Q}}_{\mathbf{b}}\equiv-\partial\bar{\mathbf{Q}}/\partial\mathbf{b}$ are derivatives of the projected vector of generalized forces $\bar{\mathbf{Q}}$; the term $\bar{\mathbf{M}}_{\mathbf{z}}\ddot{\mathbf{z}}\equiv\bar{\mathbf{M}}_{\mathbf{z}}\otimes\ddot{\mathbf{z}}$ is a tensor-vector product that represents the derivatives of the vector $\bar{\mathbf{M}}\ddot{\mathbf{z}}$ considering $\ddot{\mathbf{z}}$ as a constant. This technique results in one system of ODEs per design parameter, and is also referred to as the tangent linear model (TLM). These sets of ODEs can be integrated together with the set of ODEs in Eq. (1) for the independent sensitivities, $\mathbf{z}_{\mathbf{b}}$.

Manual vs. automatic differentiation approach

The derivatives necessary to construct the TLM are not easy to obtain. Nevertheless, they can be systematically implemented in the multibody software, in a completely general manner, as a set of additional modules on top of the usual forward dynamics modules. The resultant software is able to evaluate both Eqs. (1) and (2) without any further approximation, other than the numerical integrator employed to solve the ODE systems. In this work, this scheme is referred to as the *manual differentiation* (MD) approach.

Automatic or *algorithmic differentiation* (AD) is an alternative computational-mathematical technique for the differentiation of computer functions [3]. It is based on the decomposition of computer routines in elementary arithmetic operations (addition, subtraction, product, division) and calls to library functions (sine, cosine, exponential, etc.). These computational graphs are then used to apply the chain rule of differentiation systematically. This way, instead of manually computing the terms in Eq. (2), AD is used to directly differentiate Eq. (1) and obtain acceleration sensitivities, $\ddot{\mathbf{z}}_{\mathbf{b}}$. In first-order form [4]:

$$\dot{\mathbf{y}}_{\mathbf{b}} = \dot{\mathbf{y}}_{\mathbf{b}}(t,\mathbf{y},\mathbf{y}_{\mathbf{b}},\mathbf{b}) = \frac{d}{d\mathbf{b}} \left\{ \begin{array}{c} \dot{\mathbf{q}} \\ \bar{\mathbf{M}}(\mathbf{z},\mathbf{b})^{-1}\bar{\mathbf{Q}}(t,\mathbf{z},\dot{\mathbf{z}},\mathbf{b}) \end{array} \right\} \tag{3}$$

where state vector $\mathbf{y}^T \equiv \left\{\mathbf{q}^T, \dot{\mathbf{z}}^T\right\}$ has been defined.

Sensitivity analysis of a coach maneuver

In order to assess numerical efficiency, a coach is simulated while performing a double lane-change maneuver, and the aforementioned methods are used to compute its design sensitivities. The coach under study is a Noge Touring 345 vehicle with frame from Mercedes-Benz. A coordinate-measuring machine has been used on the unloaded coach to obtain global dimensions and the position of key suspension points and joints. A general view of the real coach is shown in Fig. 1. The coach has two axles: the front one has two wheels and the rear one four (assembled as two sets of dual wheels). The total mass of the coach is 17,048 kg when loaded.

Figure 1: Coach dynamic maneuver.

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