Second Order Sensitivities of the Dynamic Response of Multibody Systems with Penalty Formulations.

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Abstract

Sensitivity analysis of the dynamics of multibody systems is essential for design optimization. Dynamic sensitivities, when needed, are often calculated by means of finite differences but, depending of the number of parameters involved, this procedure can be very demanding in terms of time and the accuracy obtained can be very poor in many cases if real perturbations are used. In previous works [1] the authors developed the direct differentiation sensitivity equations and the adjoint variable sensitivity equations for penalty formulations to obtain the first order sensitivities of the dynamic response of multibody systems. The aim of this work is to extend the previous formulations to obtain the second order sensitivities using both the direct and the adjoint methods. The calculation of the second other sensitivities is quite cumbersome but it provides important information for both sensitivity analysis and optimization of multibody systems.

The equations of motion (EOM) of the penalty formulation presented in [2], have the following expression

$$\mathbf{M}\ddot{\mathbf{q}}+\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\alpha\left(\ddot{\mathbf{\Phi}}+2\xi\omega\dot{\mathbf{\Phi}}+\omega^2\mathbf{\Phi}\right)=\mathbf{Q}, \tag{1}$$

where $\mathbf{q}\in\mathbb{R}^n$ is the vector of coordinates of the system, $\boldsymbol{\rho}\in\mathbb{R}^p$ is the vector of parameters, $\mathbf{M}(\mathbf{q},\boldsymbol{\rho})\in\mathbb{R}^{n\times n}$ is the mass matrix of the system, $\mathbf{Q}(\mathbf{q},\dot{\mathbf{q}},t,\boldsymbol{\rho})\in\mathbb{R}^n$ is the vector of generalized forces of the system, $\mathbf{\Phi}(\mathbf{q},t,\boldsymbol{\rho})\in\mathbb{R}^m$ is the vector of constraints that relate the dependent coordinates, α is the penalty factor and ξ,ω are coefficients of the method.

Note that the EOM (1) depend on some design parameters $\boldsymbol{\rho}\in\mathbb{R}^p$ (typically masses, lengths, or other parameters related to forces chosen by the engineer) by means of the mass matrix, generalized forces and constraints. Therefore $\mathbf{q}=\mathbf{q}(t,\boldsymbol{\rho})$, $\dot{\mathbf{q}}=\dot{\mathbf{q}}(t,\boldsymbol{\rho})$, $\ddot{\mathbf{q}}=\ddot{\mathbf{q}}(t,\boldsymbol{\rho})$ too.

Let's assume the following objective function dependent on the parameters and states of the system,

$$\psi=w(\mathbf{q}_F,\dot{\mathbf{q}}_F,\boldsymbol{\rho})+\int_{t_0}^{t_F}g(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\rho})\,\mathrm{dt}. \tag{2}$$

The gradient of the objective function Eq.(2), $\nabla_{\boldsymbol{\rho}}\psi$, depends on the parameters explicitly and also implicitly by means of the dynamic response of the system $\mathbf{q}=\mathbf{q}(t,\boldsymbol{\rho})$, $\dot{\mathbf{q}}=\dot{\mathbf{q}}(t,\boldsymbol{\rho})$.

Two different methods have been traditionally employed [3] to obtain sensitivities of objective functions with respect to parameters: the direct differentiation method and the adjoint variable method.

In the direct differentiation method the gradient directly obtained as,

$$\nabla_{\boldsymbol{\rho}}\psi^{\mathrm{T}}=\frac{\mathrm{d}\psi}{\mathrm{d}\boldsymbol{\rho}}=\frac{\partial w}{\partial\mathbf{q}_F}\frac{\partial\mathbf{q}_F}{\partial\boldsymbol{\rho}}+\frac{\partial w}{\partial\dot{\mathbf{q}}_F}\frac{\partial\dot{\mathbf{q}}_F}{\partial\boldsymbol{\rho}}+\frac{\partial w}{\partial\boldsymbol{\rho}}+\int_{t_0}^{t_F}\left(\frac{\partial g}{\partial\mathbf{q}}\frac{\partial\mathbf{q}}{\partial\boldsymbol{\rho}}+\frac{\partial g}{\partial\dot{\mathbf{q}}}\frac{\partial\dot{\mathbf{q}}}{\partial\boldsymbol{\rho}}+\frac{\partial g}{\partial\boldsymbol{\rho}}\right)\mathrm{dt} \tag{3}$$

In the adjoint variable method the gradient is indirectly obtained as [4],

$$\mathscr{L}(\boldsymbol{\rho})=\psi-\int_{t_0}^{t_F}\boldsymbol{\mu}^{\mathrm{T}}\left(\bar{\mathbf{M}}(\mathbf{z},\boldsymbol{\rho})\ddot{\mathbf{z}}-\bar{\mathbf{Q}}(t,\mathbf{z},\dot{\mathbf{z}},\boldsymbol{\rho})\right)\mathrm{dt} \tag{4a}$$

$$\nabla_{\boldsymbol{\rho}}\psi=\nabla_{\boldsymbol{\rho}}\mathscr{L} \tag{4b}$$

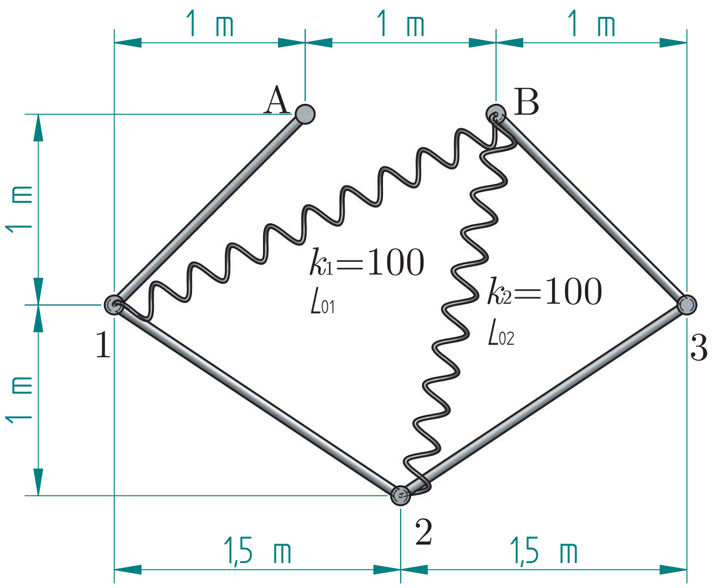

Figure 1: Five bar mechanism

The theoretical derivation and practical implementation of equations (3) and (4) in a fully general and systematic manner, is not straightforward at all for the penalty equations Eq. (1) and it was developed in [1].
In this work the extension of the sensitivity eoquations to calculate the second order sensitivities $\nabla^2_{\boldsymbol{\rho\rho}}\psi$ from (3) and (4) under a general framework is explored and the results are applied to the optimization of the dynamic response of a 2 degrees of freedom five-bar mechanism, using first and second order sensitivities.

References

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