Assessment of linearization approaches for multibody system dynamics

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The dynamics of multibody systems is frequently expressed by means of a set of n generalized coordinates $\mathbf{q} = [q_1, \ldots, q_n]^{\mathrm{T}}$ subjected to m kinematic constraints $\mathbf{\Phi} = \mathbf{0}$. For the purposes of this study, we assume that the m constraints are independent. The corresponding dynamics equations are often a highly nonlinear system of Differential Algebraic Equations (DAEs), which can be expressed in the form

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{f} + \mathbf{f}_c \tag{1a}$$

$$\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0} \tag{1b}$$

where $\mathbf{M}$ is the $n \times n$ mass matrix, $\mathbf{f}$ the term of generalized forces, and $\mathbf{f}_c$ the reactions introduced by the constraints. However, some applications like estimation and control can benefit from the use of linearized models to represent the system dynamics.

The constrained dynamics problem in Eqs. (1) can be reformulated according to one of the following choices of coordinates:

1. minimal coordinate set (MCS), with as many coordinates as degrees of freedom the system has, $n-m$;
2. redundant coordinate set (RCS), with n coordinates and a set of m Lagrange multipliers; and
3. unconstrained coordinate set (UCS), with n coordinates and some method to embed the constraints in the formulation.

Projection methods that reduce the problem to the tangent subspace of the constraints [1] are an example of the MCS approach. Baumgarte stabilization [2] can be used to represent the system dynamics in both RCS and UCS form. Expressing the dynamics with a UCS description can be achieved with several other approaches, e.g. the penalty and the augmented Lagrangian ones in [3], and the force projection approach proposed in [4]. Often, the choice of the coordinate set, and of the way the problem is subsequently formulated, is made according to the characteristics of the problem; for example, projection provides a compact differential problem when applied to rigid mechanisms, since $n-m \ll n$, whereas it provides limited problem size reduction when applied to deformable mechanisms, since in such cases $n-m \approx n$. Depending on the selected strategy, the linearization of the dynamics equations can be performed in different ways.

Linearization after projection directly yields a problem with the spectrum of the constrained system. Generalized eigenanalysis, generalized singular value decomposition and generalized Schur decomposition applied to the linearization of Eqs. (1) can be used to isolate the spectrum of the problem [5]. If the problem is expressed with the UCS choice, the related spectrum is expected to contain $2(n-m)$ eigenvalues that either are the exact spectrum of the constrained dynamics problem, or an approximation of it. It also contains $2m$ spurious eigenvalues, whose identification depends on the approach used to formulate the constrained dynamics problem.

We intend to evaluate these different approaches in terms of their suitability for the linearization of multibody dynamics. The linearized dynamics equations may describe the exact constrained dynamics or yield approximated, modified dynamics. In the latter case a study of the factors that influence the accuracy of the method is necessary. Moreover, as mentioned, some representations result in the introduction of dynamics components that do not

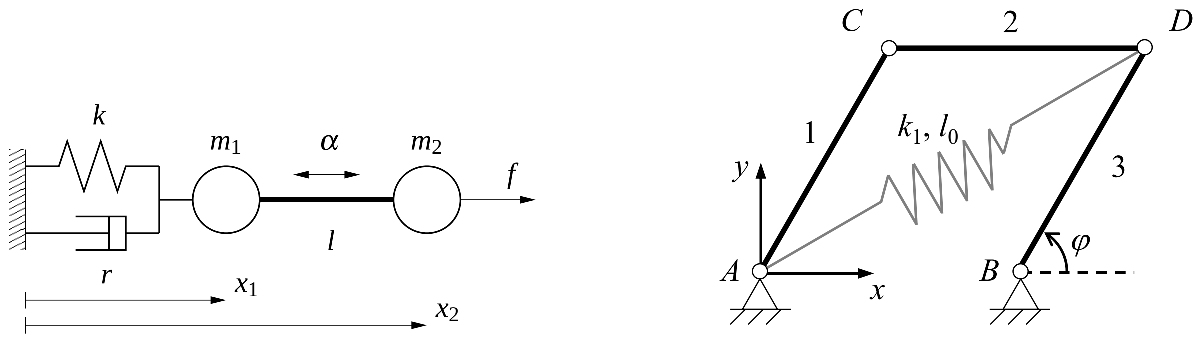

Fig. 1: Test problems: two-mass assembly (left) and four-bar linkage (right)

correspond to the physical behaviour of the mechanical system. It is desirable that such spurious dynamics be easily identifiable. Finally, a study of the efficiency of the different formulations is required. Our understanding of efficiency comprises several aspects, which include the ease of formulating and implementing the method, the speed in delivering the solution for a required level of accuracy, and the use of computational resources. The results of this study can provide guidelines for the selection of a particular formulation and linearization technique to suit the characteristics of a given problem.

Two simple benchmark problems (Fig. 1) were selected and used to obtain preliminary results with several linearization methods. The first one is a one-degree-of-freedom, double-mass and spring system described and studied in [6], [5], and [7]. The two masses are connected by a rigid-body constraint. The second one is a one-degree-of-freedom four-bar linkage composed of rods of length l with a uniformly distributed mass m_3, moving under gravity effects. A spring with stiffness k_1 and natural length l_0 connects the two tips of the diagonal $A-D$.

The linearized dynamics of both systems were obtained with different methods, which included projection on the subspace of admissible motion, Baumgarte stabilization, penalty formulations, use of generalized eigenvalues, and generalized Schur (QZ) decomposition. The results obtained with these examples will be further substantiated with the study of more complex, real-life applications.

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