Benchmark Problems for the Linearization of Multibody Dynamics

Francisco González[1], Bruce Minaker[2], Pierangelo Masarati[3],
Alberto Luaces[1], David Vilela[1], Javier Cuadrado[1]

[1]Laboratorio de Ingeniería Mecánica
University of La Coruña
Mendizábal s/n, 15403 Ferrol, Spain
[f.gonzalez, aluaces,
david.vilela]@udc.es, javicuad@cdf.udc.es

[2] Mechanical, Automotive, & Materials Engineering
University of Windsor
401 Sunset Avenue, Windsor, ON, Canada
bminaker@uwindsor.ca

[3] Dipartimento di Scienze e Tecnologie Aerospaziali
Politecnico di Milano
via La Masa 1, 20156, Milano, Italy
pierangelo.masarati@polimi.it

Abstract

Multibody formulations express the dynamics of a mechanical system in terms of a set of n generalized coordinates $\mathbf{q}$ and their derivatives. These may be independent; alternatively they can be related by m kinematic constraint equations $\mathbf{\Phi}(\mathbf{q},t) = \mathbf{0}$. In the latter case, the dynamics is expressed by a system of Differential Algebraic Equations (DAEs)

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{f} + \mathbf{f}_c \tag{1a}$$

$$\mathbf{\Phi}(\mathbf{q},t) = \mathbf{0} \tag{1b}$$

where $\mathbf{M}$ is the $n \times n$ mass matrix, $\mathbf{f}$ comprises the applied and velocity-dependent forces, and $\mathbf{f}_c$ stands for the constraint reactions. If the coordinates $\mathbf{q}$ are independent, then the dynamics is described by Eq. (1a) alone with $\mathbf{f}_c = \mathbf{0}$, which is a system of Ordinary Differential Equations (ODEs).

A wide variety of coordinate selections, formulations, and numerical methods to deal with the solution of such systems have been proposed during the last decades [1], which has prompted the multibody dynamics community to put forward a significant number of benchmark problems [2]. These benchmark problems serve two purposes. First, they allow researchers to validate newly proposed methods or software implementations. Second, they can be used as a means to compare the efficiency, ease of use, and accuracy of different algorithms and codes, providing useful information for the selection of the most appropriate ones for a given application.

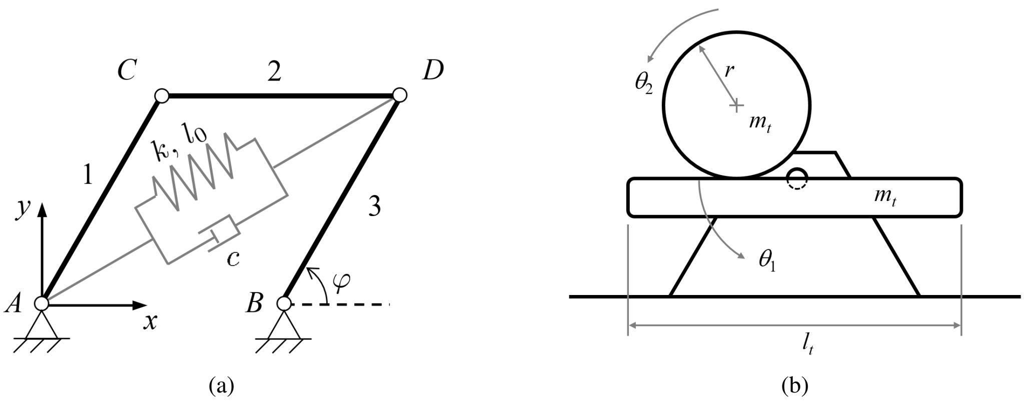
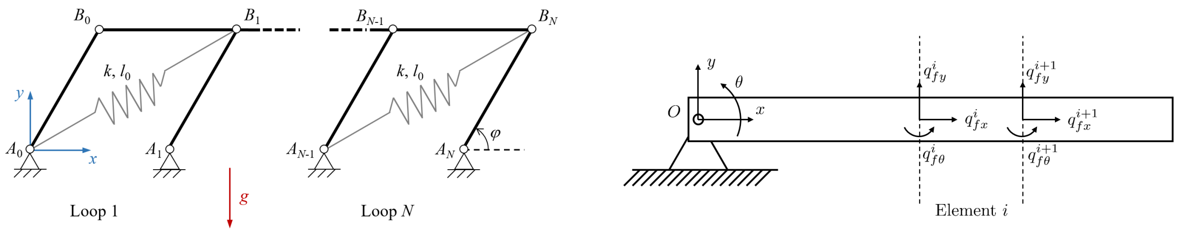


Figure 1: Two of the proposed benchmark problems: (a) Four-bar linkage with spring and damper elements; (b) A wheel rolling on a tipping table.

Regardless of the approach followed to obtain Eqs. (1), the resulting system is highly nonlinear in most cases. However, some applications such as modal analysis require a linearized expression of the dynamics. In others, like stability analysis, the availability of a linear model makes it simpler to gain insight into the system behaviour. The linearization of Eqs. (1) can be achieved in many different ways and several approaches have already been published in the multibody literature. The properties of each approach depend on the coordinate selection and the way in which kinematic constraints are treated [3]. Each method has different accuracy and efficiency properties and conveys information about the system dynamics in its own particular way.

In this work, we introduce a series of test problems to benchmark linearization methods for multibody dynamics. Some of these have already been added to the IFToMM library of Computational Benchmark Problems [2]. The examples can be categorized into two main groups:



(a) (b)

Figure 2: Benchmark problems with variable dimensions: (a) N-loop four-bar linkage; (b) A flexible pendulum modelled with finite elements.

- Simple problems with known analytical solutions (Fig. 1): a simple rigid pendulum, a four-bar linkage with spring and damper elements, a wheel rolling on a tipping table.
- Variable-size problems that can be used to assess the computational efficiency of the methods. Among these, two kinds of problems can be distinguished: heavily constrained systems with relatively few degrees of freedom, e.g., the N-loop version of the four-bar linkage in Fig. 2a, in which the number of kinematic constraints is similar to system size ($m \approx n$); and systems with many more variables than constraints ($m \ll n$), such as flexible beams and pendulums (Fig. 2b).

The proposed test problems have been used to validate and compare the performance of three multibody linearization methods based on velocity transformations, direct eigenanalysis, and a penalty formulation respectively [3]. Results showed that these methods were able to match the theoretical solutions of the first type of problems. The linearization of the proposed examples highlighted the differences between the three approaches in terms of computational efficiency and accuracy of the obtained eigenvalues.

Acknowledgments

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