# Using Kinematic Rolling Surfaces for Fast Foot-Ground Modeling in the Forward Dynamics of Human Gait — A Sagittal Plane Analysis

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## Abstract

Modeling of the foot-ground interaction is a topic of increasing interest for forward dynamics simulations of human gait, as it is essential for biofidelic and fast codes. Currently, most approaches use arrays of soft spheres or ellipsoids attached to a hind- and forefoot rigid body, interconnected by a revolute metatarsal joint, (e.g. [1], [2], [3]. This is accurate enough but (a) requires significant computational effort to find equilibrium configurations, and (b) induces superfluous high-frequency oscillations of the foot segments with respect to each other and the ground, both slowing down forward dynamics integration schemes. Recently, an alternative approach describing the rolling behavior of the foot by a surrogate disk with exponentially decaying radius as a function of foot tilt angle was presented [4], and a more or less recurrent kinematical rolling behavior of foot-ground interaction in the sagittal plane was verified in experiments for a large portion of the foot contact (see Fig. 1 (b), displaying average tilt angle over CoP progression (blue curve) and its standard deviation for 7 healthy walkers, as well as average curvature radius (red curve) and its standard deviation over CoP progression) [5]. Thus one may regard the foot-ground interaction as a higher joint from where individual motions will depart by small perturbations. This is analyzed in this paper for a simple forward dynamics period during the sagittal stance phase.

The disk-ground contact is parametrized by a virtual contact disk with exponentially decaying radius $r(\alpha) = A(1-e^{-C|\alpha|})$ whose rim touches the ground without slip at the immaterial contact point $P$ (Fig. 1 (a)) [4], where $A, C$ are shaping parameters. From this the physical rolling point (coinciding with the CoP) can be determined as follows: Let $r^\star$ be the distance of the immaterial contact point $P$ from the footprint center $C^\star$ corresponding to $P$ for $\alpha = 0$. For an infinitesimal increase $\mathrm{d}\alpha$, point $P$ progresses by $\mathrm{d}r^\star = r' \cos\alpha \, \mathrm{d}\alpha$ outwards, where $(\cdot)' = \partial/\partial\alpha$ and $\mathrm{d}r^\star$ is the projection of $\mathrm{d}r$ on the ground. The material rolling point $\Omega$ currently having velocity zero must be at a distance $\widehat{r}^\star$ from the point $C^\star$ such that the vertical velocity component $\dot{z}_a = \mathrm{d}\{r(\alpha)\sin\alpha\}/\mathrm{d}t$ of the virtual disk center is equal to its vertical roll velocity component $[\widehat{r}^\star - (r^\star - r\cos\alpha)]\dot{\alpha}$. Thus, one obtains

$$\widehat{r}^\star = r^\star + r'\sin\alpha \quad , \quad \text{with} \quad r^\star(\alpha) = \int_0^\alpha r'(\bar{\alpha})\cos\bar{\alpha}\,\mathrm{d}\bar{\alpha} = \frac{AC}{1+C^2}\left[\sin\alpha\, e^{-C|\alpha|} + C\left(1-\cos\alpha\, e^{-C|\alpha|}\right)\right] . \quad (1)$$

Note that the exponential radius approach renders an explicit function for the roll distance in terms of the tilt angle $\alpha$, while ellipsoids require elliptic integrals for this purpose. In order to allow for more generic rolling surface shapes, a linear combination of several virtual exponential radius terms can be used such as

$$r(\alpha) = \sum_{i=1}^{N} A_i\left(1-e^{-C_i|\alpha|}\right) < \mathrm{switch}\{\alpha_i^\star\} > \quad (2)$$

where "switch" turns on/off the individual terms depending on the angle $\alpha_i^\star$. In the present case $i = 1,2$ are turned on *before*, and $i = 3,4$ are turned on *after* the switching angle $\alpha^\star = 0.0260$. The other shaping parameters where chosen such that (1) curvature radius and its derivative (=0) at $\alpha = 0$ correspond to the average curve; (2) curvature radius and rolling distance at the interface $\alpha^\star$ match for the two pairs of exponential radius functions; and (3) the rest of the curvature progression matches as close as possible the average of measurement. The corresponding

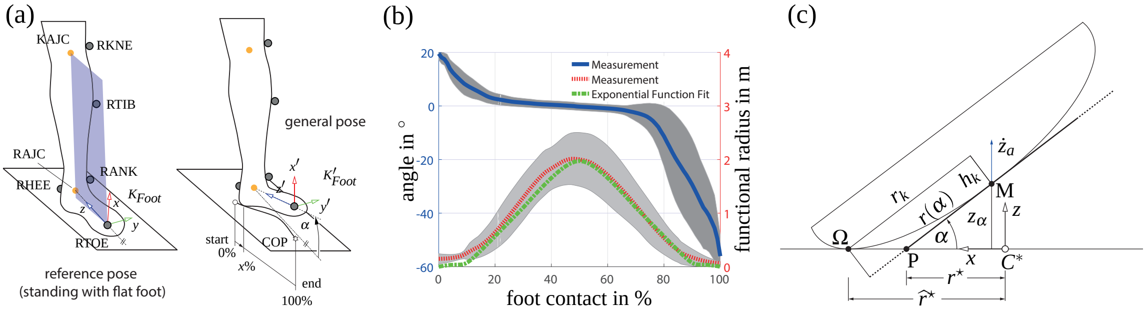
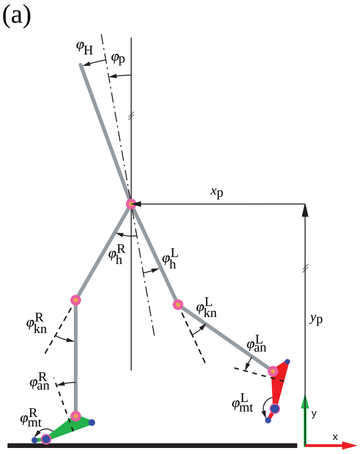
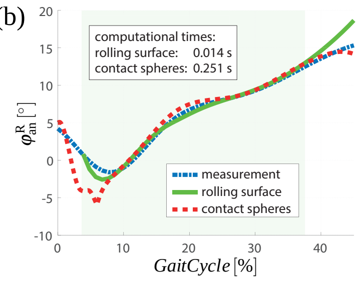



Figure 1: (a) definition of foot inclination angle $\alpha$ and CoP progression $x$ as percentage of total footprint length (b) experimental foot rolling results of 7 healthy walkers (c) exponential radius rolling surface in the sagittal plane.

(a)



(b)




| $A_1$ | $C_1$ | $A_2$ | $C_2$ |
|---|---|---|---|
| 0.0665 | 17.0 | $8.86 \cdot 10^{-4}$ | 147.26 |
| $A_3$ | $C_3$ | $A_4$ | $C_4$ |
| 0.0515 | 2.0 | 0.0548 | 18.0 |

Table 1: Parameters for Eq. (2) fit

| parameter | value | | |
|---|---|---|---|
| $m_{\text{total}}$ | $65\,kg$ | $k_{\text{an,L}}$ | $25.0\,N/m$ |
| $\ell_{\text{HAT}}$ | $0.556\,m$ | $k_{\text{s1,heel}}$ | $6.5 \cdot 10^5\,N/m$ |
| $\ell_{\text{Femur}}$ | $0.417\,m$ | $k_{\text{s2,mt}}$ | $1.0 \cdot 10^9\,N/m$ |
| $\ell_{\text{Tibia}}$ | $0.435\,m$ | $k_{\text{s3,tip}}$ | $1.0 \cdot 10^9\,N/m$ |
| $\ell_{\text{Footx}}$ | $0.182\,m$ | $e_{\text{s1,heel}}$ | 0.4 |
| $\ell_{\text{Footy}}$ | $0.043\,m$ | $e_{\text{s2,mt}}$ | 0.2 |
| $\ell_{\text{toe}}$ | $0.05\,m$ | $e_{\text{s3,tip}}$ | 0.2 |
| $r_{\text{s1,heel}}$ | $0.025\,m$ | $\mu_{\text{st,heel}}$ | 0.5 |
| $r_{\text{s2,mt}}$ | $0.029\,m$ | $\mu_{\text{st,mt}}$ | 0.9 |
| $r_{\text{s3,tip}}$ | $0.023\,m$ | $\mu_{\text{st,tip}}$ | 0.9 |

Table 2: Sphere-contact parameters

Figure 2: (a) Schematic two-dimensional multibody model from [6], (b) plot of the right ankle angle as a result of a forward dynamic simulation comparing viscoelastic spheres and virtual exponential disk with measurement data.

values are listed in Table 1, and the resulting curvature curve is shown in Fig. 1 (b) in green. Note that a fairly good fit is achieved by only nine parameters (in fact of which only 5 are independent).

Forward dynamics was analyzed for a simple two-dimensional biomechanical multibody walking model in the sagittal plane as presented in [6] (Fig. 2 (a)) using both viscoelastic spheres with Hunt-Crossley damping as in [2], and the here presented exponential rolling surface without metatarsal joints. Basic assumptions of the model were: (i) the pelvis is hinged to the inertial system via two prismatic joints ($x_{\text{p}}$, $y_{\text{p}}$) and one revolute joint ($\varphi_{\text{p}}$); (ii) head and torso (HAT) are reduced to one rigid body connected by an ideal revolute joint ($\varphi_{\text{H}}$) to the pelvis; and (iii) legs are chains of revolute joint/rigid link pairs starting at the hip-thigh joint ($\varphi_{\text{h}}$) and followed by knee-shank, ankle-hindfoot, and metatarsal joint-forefoot ($\varphi_{\text{kn}}, \varphi_{\text{an}}, \varphi_{\text{mt}}$). For the forward dynamics study, a real gait motion was first tracked and then a simulation with hybrid joint actuation comprising identical inputs for both models as (a) kinematically-driven (=rheonomic constraint) joints comprising $\varphi_{\text{H}}$, $\varphi_{\text{h}}$ and $\varphi_{\text{kn}}$ for both legs as well as $\varphi_{\text{an}}$ for the swing leg, and computed-torque controlled joints comprising $x_{\text{p}}$, $y_{\text{p}}$, $\varphi_{\text{p}}$, $\varphi_{\text{an}}$ of the stance leg, as well as both metatarsal joints for the viscoelastic sphere model. For the rolling surface case, simulation was started from the point of contact, as no impact is regarded at this point of the development. Due to the kinematical constraint, the rolling surface model has three degrees of freedom less than the viscoelastic sphere model, and the kinematics of the closed loop resulting from pelvis motion and kinematic foot rolling joint was solved numerically by Newton iterations. Fig. 2 (b) shows the resulting simulation results for the stance-foot ankle joint for the viscoelastic spheres (red dashed) and the virtual exponential disk (green solid). Compared to the measurement (blue dotted line), one can appreciate that the kinematic rolling surface model renders comparably good results — if not better — than the sphere contact between 3% and 35% of the gait phase, while performing approx. 18 times faster. The deviation after 35% is due to the missing foot contact of the opposite leg and the missing metatarsal joint torque control, which can be considered in future. This shows that modeling foot-ground contact by kinematic rolling surfaces might be an interesting alternative to soft sphere contacts for fast forward dynamics simulation of human gait.

Further work will be devoted to extend the virtual contact disk model to general spline fits, regularized impacts for foot strike, tangential compliance for soft foot pad deflection, as well as rotations also in the frontal plane.

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