Kinematic consistency of position analysis in the push-up exercise from experimental 3D markers data

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Abstract

Physical exercises based on loads applied to the body by means only of gravity and reaction forces and moments have many advantages compared to bodybuilder devices. One of the most popular exercises of this class is the push-up, used for strengthening the upper part of the body. The biomechanics of this task is complex, involving multiple closed-loop chains and full body motion. The shoulder is the most important body part of the exercise, and its modeling involves four bones and many muscles with complex geometry.

This task has been modeled by means of a system with 21 bodies (Figure 1), each with 12 natural coordinates, resulting into \( n=252 \) coordinates, which are constrained by \( m=214-6=208 \) (6 redundant) algebraic equations, resulting \( f=n-m=44 \) degrees of freedom.

The reflexive marker's movement of 10 healthy subjects performing push-ups was recorded by a BTS Smart-D Motion Capture System (BTS Bioengineering, Italy) at 500 Hz of sample frequency. The coordinates were low-pass filtered. Ground reaction forces on each foot and hand were measured by BTS P-6000 force platforms. The marker set and guidelines to calculate the local coordinate system of each body segment followed the International Society of Biomechanics recommendations. The natural coordinates were calculated from the Cartesian coordinates of the markers positioned over anatomical landmarks. However, errors from the
motion analysis system and from relative bone to skin artifacts turn the coordinates kinematically inconsistent [1]. To guarantee that the linkage position analysis was consistent the following optimization problem was formulated:

$$\min f(q) = \frac{1}{2} (q - q^*)^T W (q - q^*)$$

subject to $\phi(q) = 0$  \hspace{1cm} (1)

where $q$ is the set of kinematically consistent natural coordinates $[n \times 1]$, $q^*$ is the raw, kinematically inconsistent coordinates set, $W$ is the diagonal weighting matrix $[n \times n]$, and $\phi(q)$ is the vector of constraint equations $[m \times 1]$. The optimization problem was solved by the augmented Lagrangian minimization process [2]:

$$\left(W + \phi_q^T \lambda \right) \Delta q_{i+1} = -W(q_i - q^*) - \phi_q^T (\alpha \lambda + \lambda_i)$$

$$\lambda_{i+1} = \lambda_i + \alpha \lambda$$  \hspace{1cm} (2)

where $\Delta q_{i+1} := q_{i+1} - q_i$; $\phi_q$ is the Jacobian matrix of the vector of constraint equations $[m \times n]$; $\lambda$ is the vector of Lagrange multipliers $[m \times 1]$; and $\alpha$ is the penalty factor diagonal matrix $[n \times n]$, with scalar values $\sim 10^7$. ‘Temporary’ constraints were formulated to assure that the hands and feet were always in contact with the floor.

$$\phi_{temp} = C_{CP} q - C_P$$  \hspace{1cm} (3)

$C_P$ is the center of pressure position vector given by the force plate and $C_{CP}$ is the matrix that transforms the generalized coordinates in the center of pressure position vector. The constraint vector norm ($|\phi|$) is shown in Figure 2, demonstrating that all constraints were fulfilled throughout a series of five push-ups for one sample subject. The reconstructed kinematics patterns were consistent with the observed movement. Variations relatively to the kinematical model complexity, e.g. considering only glenohumeral joint for the shoulder, were also addressed.

![Figure 2. Norm of the constraint vector after the kinematic consistency.](image)

References
