INTERFACING MULTIBODY DYNAMICS STEPPING FORMULATIONS IN
CO-SIMULATION SETUPS

Albert Peiret, Francisco González, József Kövecses
Dept. of Mechanical Engineering & Centre for Intelligent Machines
McGill University, Montréal, Québec H3A 0C3, Canada
Email: albert.peiret@mail.mcgill.ca, franglez@cim.mcgill.ca, jozsef.kovecses@mcgill.ca

Marek Teichmann
CM Labs Simulations, Inc.
Montréal, Québec H3C 1T2, Canada
Email: marek@cm-labs.com

ABSTRACT

In numerous engineering applications, mechanical systems interact with external components of a different physical nature, such as hydraulics and electronics. The simulation of such systems can be carried out in an efficient and modular way by means of non-iterative co-simulation schemes, in which each subsystem is integrated separately by its own solver, and coupling variables are exchanged at discrete communication time points. This approach, however, may become unstable for large communication step-sizes, especially when the dynamics of the subsystems are not smooth.

Modelling the interface between the mechanical system and other components in the assembly with a reduced order model (ROM) can be used to increase the communication step-size and improve the numerical stability of the simulation. Here, we introduce the expression of a ROM for the co-simulation of non-smooth mechanical systems subjected to contacts and friction.

INTRODUCTION

Time-stepping algorithms are a robust and efficient way to carry out the forward-dynamics simulation of multibody systems subjected to unilateral and bilateral constraints. For a multibody system \( M \) in a co-simulation setup, described by \( n \) generalized velocities \( \mathbf{v} \) and subjected to \( m \) kinematic constraints \( \mathbf{w}_c = \mathbf{A}_c \mathbf{v} \), the time-stepping formulation can be expressed as

\[
\begin{bmatrix}
M - A_c^T A_c & -A_c^T \\
A_c & C_c
\end{bmatrix}
\begin{bmatrix}
\mathbf{v}^+ \\
h\mathbf{\lambda}_c^+
\end{bmatrix}
+ \begin{bmatrix}
-M \mathbf{v} - h \mathbf{f} \\
\mathbf{d}_c
\end{bmatrix}
= \begin{bmatrix}
0 \\
\mathbf{w}_c^+
\end{bmatrix}
\tag{1}
\]

where \( h \) is the integration step-size, and \( \mathbf{v} \) and \( \mathbf{v}^+ \) are the velocity at the beginning and at the end of the time-step, respectively. \( \mathbf{\lambda}_c \) are the constraints reactions, and the diagonal matrix \( C_c \) and the vector \( \mathbf{d}_c \) account for the constraint regularization. The generalized applied forces \( \mathbf{f} = \mathbf{f}_0 + A_c^T \mathbf{\lambda}_i \) include the forces \( \mathbf{\lambda}_i \) exchanged at the co-simulation interface, which is parametrized by \( p \) interface velocities \( \mathbf{w}_i = \mathbf{A}_i \mathbf{v} \).

We couple the dynamics in (1) with another subsystem \( S \) of a different nature, e.g., hydraulic actuators. This is done via the exchange of the interface force \( \mathbf{\lambda}_i \) (as input of the multibody system \( M \)) and the interface velocity \( \mathbf{w}_i \) (as input of the subsystem \( S \)). Unfortunately, due to the nature of such subsystems, they need to be integrated at a much higher time rate than the multibody system. In co-simulation setups, interface variables are only exchanged at discrete communication time points, and so the evolution of the input variables between exchange points needs to be handled (using extrapolation techniques, for instance), which typically results in unstable numerical integration due to discontinuities in the subsystem inputs [1]. Instead, we propose a reduced order model (ROM) of the multibody system to represent its dynamics within the macro time-step of size \( h \).
REDUCED ORDER INTERFACE MODELS

A reduced order model (ROM) can give the possibility to predict the behaviour of the multibody system \( \mathcal{M} \) within a macro time-step. Hydraulics and electric subsystems usually have faster dynamics than mechanical ones, and so they require shorter integration step-sizes. This means that several steps may be taken by their solvers before the inputs that they receive from the mechanical system are updated at the next communication point. These inputs can then be kept unchanged at their last known value, or may be extrapolated from previously stored ones. In non-smooth systems, this often leads to unstable behaviours. The ROM approach, on the other hand, makes use of effective mass and force terms that represent subsystem \( \mathcal{M} \), which makes it possible to approximate its dynamics until the next communication point. The concept was introduced in [2], where the expression of a ROM for the acceleration-level dynamics equations of multibody systems was provided as well.

When a time-stepping algorithm like the one in Eqs. (1) is used to solve the dynamics of a multibody system, the dynamics of the corresponding ROM within the macro time-step in terms of the input force \( \lambda_i^+ \) can be given by

\[
\tilde{M}(w_i^\alpha - w_i^0) = \alpha h(\lambda_i^+ + \tilde{f})
\]  

(2)

where \( \alpha \in [0, 1], w_i^0 = A_i(I - P_c)v \) represents the interface velocity at the beginning of the step, and \( w_i^\alpha \) is the interface velocity at any time-point within the step. The effective mass and force terms can be written as

\[
\tilde{M} = (A_i(I - P_c)M^{-1}A_i)^{-1} \\
\tilde{f} = MA_i(I - P_c)M^{-1}f_0
\]  

(3)

(4)

and the projector matrix is

\[
P_c = M^{-1}A_c(A_cM^{-1}A_c^T + C_c)^{-1}A_c
\]  

(5)

The effective terms in Eqs. (3) and (4) are evaluated at the beginning of each macro step and used to integrate the ROM dynamics until the next communication point. This can be used to provide the non-mechanical subsystems in the co-simulation environment with a physics-based prediction of the evolution of the inputs that they receive from subsystem \( \mathcal{M} \).

Additionally, by combining Eqs. (2), (3) and (4), a more efficient computation of the interface velocity \( w_i^\alpha \) given by the ROM within the macro time-step can be performed as

\[
w_i^\alpha = w_i^0 + \alpha h(a + \tilde{M}^{-1}\lambda_i^+)
\]  

(6)

where the terms \( a = A_i(I - P_c)M^{-1}f_0 \) and \( \tilde{M}^{-1} \) can be precomputed at the beginning of each macro time-step, and the only term that is updated at each micro time-step is the interface force \( \lambda_i^+ \) coming from subsystem \( S \).

TEST EXAMPLES

Several examples were used to assess the ability of the ROM in Eq. (2) to stabilize co-simulation setups. Figure 1 shows a 2D model of a hydraulically actuated mechanism, similar to the one used as benchmark problem in [3]. A multibody model of the two-link mechanical system was interfaced to a hydraulic model of the two actuators in a non-iterative co-simulation setup. Both sub-systems had non-smooth dynamics, caused by intermittent contacts in the mechanical system and fast actuation laws in the hydraulic one. The use of a ROM improved the stability of the co-simulation process, enabling the use of longer macro-steps and reducing the computational time.

REFERENCES