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DIRECT SENSITIVITY ANALYSIS OF MULTIBODY SYSTEMS MODELED WITH RELATIVE COORDINATES USING AN AUGMENTED LAGRANGIAN FORMULATION WITH PROJECTIONS.

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ABSTRACT

Nowadays, optimal control and optimal design are two of the most interesting problems in the field of mechanical engineering, and most of the approaches include, in one way or other, a sensitivity analysis.

The sensitivity analysis of the dynamics of multibody systems constitutes an important effort in the optimization process and an important amount of the CPU time is consumed in the forward dynamic calculations. Therefore, it is convenient to use the fastest available formulations to solve the dynamics.

Numerical perturbations constitute a simple method to compute the sensitivity of the dynamics of a mechanism, but it is an expensive technique in terms of computational time, and it can lead to inaccurate results. In general, the analytical calculations are faster and have a higher accuracy, but they are particular for each formulation and a generalization is not always easy to implement. Accuracy plays an important role, since poor sensitivity accuracy will usually lead to slow convergence in the optimization process or even to non-optimal solutions.

In this work, the analytical direct sensitivity for the topological semi-recursive formulations described in [1] is derived and the resulting sensitivity formulations are validated by means of a set of examples, including a five-bar linkage and a spatial slider-crank with springs. The approaches described in this work are implemented as general sensitivity formulations in the multibody simulation library MBSLIM [2].

1 INTRODUCTION

The calculation of the dynamics of multibody systems is a well known problem, solved with different formulations, numerical integrators and coordinate systems. In [3, 4], some efficient, accurate and robust approaches were presented using an augmented Lagrangian index-3 approach with projections (ALI3-P) applied to a topological semi-recursive formalism in the context of structural integrations. Some of the most important advantages of these formulations are the exact fulfillment of the constraints in positions, velocity and acceleration, their low computational cost compared to other formulations with the same accuracy and the ability to automatically handle singular mass matrices, redundand constraints and singular configurations.

Recently, the ALI3-P topological formulation has been revisited, extended, generalized and implemented in MBSLIM, resulting in the RTDyn0_ALI3-P and RTDyn1_ALI3-P formulations. These two approaches constitute a semi-recursive solution for the dynamics of multibody systems and lead, in general, to smaller computational times than the global ALI3-P formulation in natural coordinates, specially for large branched systems. The RTDyn0_ALI3-P and RTDyn1_ALI3-P formulations only differ on the selection of the reference points used to write the equations of motion and recursive kinematics (RTDyn0 uses the center of mass of each body and RTDyn 1 the point coincident with the global reference frame origin), therefore the direct sensitivity will be derived in the context of the general formulation for any reference point, being the RTDyn0_ALI3-P and RTDyn1_ALI3-P sensitivities a particularization from the general formulation.

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2 FORWARD SENSITIVITY

Lets consider a vector of objective functions $\boldsymbol{\psi} \in \mathbb{R}^o$ expressed in terms of the relative coordinates vector $\mathbf{z} \in \mathbb{R}^n$ and its time derivatives $\dot{\mathbf{z}}, \ddot{\mathbf{z}} \in \mathbb{R}^n$, the lagrange multipliers $\boldsymbol{\lambda}^* \in \mathbb{R}^{nc}$, and a set of parameters $\boldsymbol{\rho} \in \mathbb{R}^p$. The gradient of the objective function can be expressed with the following equation:

$$\begin{aligned}\boldsymbol{\psi}' = \nabla \boldsymbol{\psi}^{\mathrm{T}} &= \left(\mathbf{w}_{\mathbf{z}}\mathbf{z}_{\boldsymbol{\rho}} + \mathbf{w}_{\dot{\mathbf{z}}}\dot{\mathbf{z}}_{\boldsymbol{\rho}} + \mathbf{w}_{\ddot{\mathbf{z}}}\ddot{\mathbf{z}}_{\boldsymbol{\rho}} + \mathbf{w}_{\boldsymbol{\lambda}^*}\boldsymbol{\lambda}^*{}_{\boldsymbol{\rho}} + \mathbf{w}_{\boldsymbol{\rho}}\right)_F \\ &+ \int_{t_F}^{t_0} \left(\mathbf{g}_{\mathbf{z}}\mathbf{z}_{\boldsymbol{\rho}} + \mathbf{g}_{\dot{\mathbf{z}}}\dot{\mathbf{z}}_{\boldsymbol{\rho}} + \mathbf{g}_{\ddot{\mathbf{z}}}\ddot{\mathbf{z}}_{\boldsymbol{\rho}} + \mathbf{g}_{\boldsymbol{\lambda}^*}\boldsymbol{\lambda}^*{}_{\boldsymbol{\rho}} + \mathbf{g}_{\boldsymbol{\rho}}\right) dt\end{aligned} \tag{1}$$

In Eq. 1 the derivatives of $\mathbf{g} \in \mathbb{R}^o$ and $\mathbf{w} \in \mathbb{R}^o$ are known and the terms $\mathbf{z}$, $\dot{\mathbf{z}}$, $\ddot{\mathbf{z}}$ and $\boldsymbol{\lambda}^*$ are the unknown variables, which can be solved, following the same scheme introduced in [5], by means of p index-3 differential-algebraic systems of equations (DAE) plus $2p$ systems of linear or non-linear equations for the sensitivity of the velocity and acceleration projections.

The system of equations for the sensitivity of the dynamics for a general semi-recursive ALI3-P formulation can be expresed as:

$$\left[\mathbf{M}^d \ddot{\mathbf{z}}'^{*\{i\}} + \mathbf{C}\dot{\mathbf{z}}'^{*\{i\}} + \bar{\mathbf{K}}\mathbf{z}'^{\{i\}} + \boldsymbol{\Phi}_{\check{\mathbf{z}}}^{\mathrm{T}} \boldsymbol{\lambda}'^{*\{i\}}\right] = \bar{\mathbf{Q}}^{\boldsymbol{\rho}} \tag{2a}$$

$$\boldsymbol{\lambda}'^{*\{i\}} = \boldsymbol{\lambda}'^{*\{i-1\}} + \boldsymbol{\alpha}\boldsymbol{\Phi}' \tag{2b}$$

Where i indicates the iteration, and:

$$\bar{\mathbf{K}} = \mathbf{M}^d_{\check{\mathbf{z}}}\ddot{\mathbf{z}}^* + \boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}\check{\mathbf{z}}}\left(\boldsymbol{\lambda}^* + \boldsymbol{\alpha}\boldsymbol{\Phi}\right) + \boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\mathbf{z}} + \mathbf{K} \tag{3}$$

$$\bar{\mathbf{Q}}^{\boldsymbol{\rho}} = \mathbf{Q}^d_{\boldsymbol{\rho}} - \mathbf{M}^d_{\boldsymbol{\rho}}\ddot{\mathbf{z}}^* - \boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}\boldsymbol{\rho}}\left(\boldsymbol{\lambda}^* + \boldsymbol{\alpha}\boldsymbol{\Phi}\right) - \boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\boldsymbol{\rho}} \tag{4}$$

$$\boldsymbol{\Phi}' = \boldsymbol{\Phi}_{\check{\mathbf{z}}}\mathbf{z}' + \boldsymbol{\Phi}_{\boldsymbol{\rho}} \tag{5}$$

In the previous expressions, the subscript $(.)_{\check{\mathbf{z}}}$ indicates a total derivative with respect to $\mathbf{z}$, the symbol $(.)'$ a total derivative with respect to $\boldsymbol{\rho}$ and the subscript $(.)_{\mathbf{x}}$ marks a partial derivative with respect to any vector $\mathbf{x}$.

The fulfillment of the constraints in velocities and accelerations implies the use of velocity and acceleration projections during the dynamics, and consequently the sensitivity of this projections must be calculated. Considering $\mathbf{P} \in \mathbb{R}^{n \times n}$ a symmetric projection matrix, the sensitivity of the iterative velocity projections, can be calculated as:

$$\begin{aligned}\left(\mathbf{P} + \varsigma\boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\check{\mathbf{z}}}\right)\dot{\mathbf{z}}'^{\{i\}} &= \mathbf{P}\dot{\mathbf{z}}'^* + \mathbf{P}'\left(\dot{\mathbf{z}}^* - \dot{\mathbf{z}}\right) \\ -\boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}\check{\mathbf{z}}}\left(\varsigma\boldsymbol{\alpha}\dot{\boldsymbol{\Phi}}\right)\mathbf{z}' &- \boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}\boldsymbol{\rho}}\left(\varsigma\boldsymbol{\alpha}\dot{\boldsymbol{\Phi}}\right) - \boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}}\left(\varsigma\boldsymbol{\alpha}\mathbf{b}^{\boldsymbol{\rho}}\right)\end{aligned} \tag{6}$$

with:

$$\mathbf{b}^{\boldsymbol{\rho}} = \left(\boldsymbol{\Phi}_{\check{\mathbf{z}}\check{\mathbf{z}}}\dot{\mathbf{z}}\right)\mathbf{z}' + \boldsymbol{\Phi}_{\check{\mathbf{z}}\boldsymbol{\rho}}\dot{\mathbf{z}} + \boldsymbol{\Phi}_{t\boldsymbol{\rho}} \tag{7}$$

Following the same procedure, the sensitivity of the acceleration projections take the form:

$$\begin{aligned}\left(\mathbf{P} + \varsigma\boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}}\boldsymbol{\alpha}\boldsymbol{\Phi}_{\check{\mathbf{z}}}\right)\ddot{\mathbf{z}}'^{\{i\}} &= \mathbf{P}\ddot{\mathbf{z}}'^* + \mathbf{P}'\left(\ddot{\mathbf{z}}^* - \ddot{\mathbf{z}}\right) \\ -\boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}\check{\mathbf{z}}}\left(\varsigma\boldsymbol{\alpha}\ddot{\boldsymbol{\Phi}}\right)\mathbf{z}' &- \boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}\boldsymbol{\rho}}\left(\varsigma\boldsymbol{\alpha}\ddot{\boldsymbol{\Phi}}\right) - \boldsymbol{\Phi}^{\mathrm{T}}_{\check{\mathbf{z}}}\left(\varsigma\boldsymbol{\alpha}\mathbf{c}^{\boldsymbol{\rho}}\right)\end{aligned} \tag{8}$$

with:

$$\begin{aligned}\mathbf{c}^{\boldsymbol{\rho}} = \left(\boldsymbol{\Phi}_{\check{\mathbf{z}}\check{\mathbf{z}}}\dot{\mathbf{z}} + \dot{\boldsymbol{\Phi}}_{\check{\mathbf{z}}}\right)\dot{\mathbf{z}}' + \left(\boldsymbol{\Phi}_{\check{\mathbf{z}}\check{\mathbf{z}}}\ddot{\mathbf{z}} + \dot{\boldsymbol{\Phi}}_{\check{\mathbf{z}}\check{\mathbf{z}}}\dot{\mathbf{z}}\right)\mathbf{z}' + \boldsymbol{\Phi}_{\check{\mathbf{z}}\boldsymbol{\rho}}\ddot{\mathbf{z}} \\ + \dot{\boldsymbol{\Phi}}_{\check{\mathbf{z}}\boldsymbol{\rho}}\dot{\mathbf{z}} + \dot{\boldsymbol{\Phi}}_{t\boldsymbol{\rho}}\end{aligned} \tag{9}$$

Although the equations presented are analog to the ones introduced in [5], the main novelty is included in the calculation and assembly of all the terms appearing in the formulation with all the generality for any type of joint, constraint and topology of the mechanism.

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