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Kalman Filters Based on Multibody Models with Colored Noise

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Abstract

In the last years, the use of estimation techniques has been expanded to multiple applications. Through a reduced set of sensors and a model of the system under study, a deeper information about the system may be achieved. The estimated variables can be later used in control algorithms or for maintenance purposes. From all the estimation techniques available, Kalman filtering stands as the most popular. The filter employs a reduced set of sensors to correct the possible differences between model and reality, increasing the reliability of the estimations.

The proper behavior of the Kalman filter relies on a certain amount of assumptions. If they are not fulfilled, the performance of the filter can decrease. The main premise is that the system under study is linear, which is not the case of most real systems. Many filters have been developed to overcome this issue, such as the extended Kalman filter (EFK) or the unscented Kalman filter (UKF). The former linearizes the model through Jacobian matrix. The latter propagates a set of deterministically chosen sample points through the system function. Another assumption is that the statistical properties of the measurement and system noise are known, which is not always true. Adaptive Kalman filters (AKF) [2] are proposed in order to estimate the properties of the noise. However, AKF usually assumes that the innovation sequence of the filter is white Gaussian noise. This assumption cannot be guaranteed if both the noise of system and measurements are not white Gaussian noise, which is frequent in certain real systems. For these situations, i.e. systems with colored noise, this works combines the Kalman filter with a shaping filter for a better characterization noise.

In the proposed approach, the Kalman filter is an indirect filter known as error-state extended Kalman filter (errorEKF). It estimates the errors on the state variables instead of the variables themselves. The state vector consists in the *error* in position, velocity and acceleration of the degrees of freedom of the system, ensuring a certain level of independence within the states. Hence,

$$\boldsymbol{x}^{\mathrm{T}} = \left[\Delta \boldsymbol{z}^{\mathrm{T}}, \Delta \dot{\boldsymbol{z}}^{\mathrm{T}}, \Delta \ddot{\boldsymbol{z}}^{\mathrm{T}}\right] \tag{1}$$

where Δz, $\Delta \dot{z}$, $\Delta \ddot{z}$ are the errors in position, velocity and acceleration of the independent coordinates (coincident with the degrees of freedom) respectively.

However, for systems with colored noise, the noise can be modeled as a first order Gauss-Markov process. For a discrete model,

$$\omega(t+1) = e^{-\frac{\Delta t}{\tau}}\omega(t) + \xi(t) \tag{2}$$

where Δt is the time interval, τ is the correlation time, ξ is white noise and ω are the low frequency terms of the system noise. In order to determine ω, the state vector is increased. Hence,

$$\boldsymbol{x}^{\mathrm{T}} = \left[\Delta z, \Delta \dot{z}, \Delta \ddot{z}, \omega_{\Delta z}, \omega_{\Delta \dot{z}}, \omega_{\Delta \ddot{z}}\right]^{\mathrm{T}} \tag{3}$$

where $\omega_{\Delta z}, \omega_{\Delta \dot{z}}, \omega_{\Delta \ddot{z}}$ represent low frequencies contribution of the system noise at position, velocity and acceleration level respectively.

The general form of the propagation equations are, therefore,

$$\boldsymbol{x}_k = \boldsymbol{F}\boldsymbol{x}_{k-1} \tag{4}$$

$$\boldsymbol{P}_k = \boldsymbol{F}\boldsymbol{P}_{k-1}\boldsymbol{F}^T + \boldsymbol{Q} \tag{5}$$

where the matrices F and Q take the form of,

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{f_x} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{W} \end{bmatrix} \tag{6}$$

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q}_z & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_\omega \end{bmatrix} \tag{7}$$

where $\boldsymbol{Q}_z$ and $\boldsymbol{Q}_\omega$ are the covariance matrices for the states and noise estimation respectively and $\boldsymbol{W} = e^{-\frac{\Delta t}{\tau}}$, which is related with the autocorrelation of the system noise. In this work, $\boldsymbol{W}$ is estimated through an on-line analysis of the correlation of the innovation sequence of the filter.
The proposed filter is applied in a simple mechanism in order to properly evaluate its performance. A four-bar linkage (Figure 1) is modeled in natural coordinates with the augmented Lagrangian index-3 (ALI3P) [1] as formulation to solve the motion of the mechanism. The maneuver consists on letting the mechanism move freely due to the action of gravity.

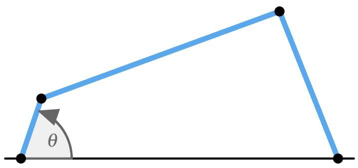
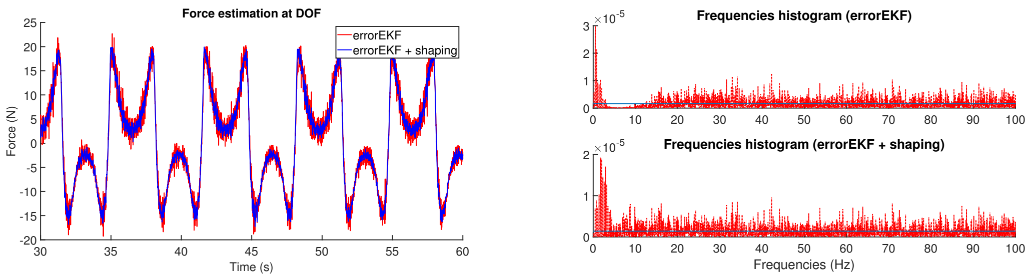


Fig. 1 Four-bar linkage.

This work is developed on a simulation environment. Hence, three multibody models are employed in order to replicate a real situation. The first one is considered as the *real mechanism* and the sensor measurements are gathered from it. The second multibody model acts as a *model* of the *real mechanism*, as it includes known modeling errors. The third model combines the *model* with the proposed filter, correcting the errors based on the measurements provided by the *real mechanism*.
The results can be seen in Figure 2a. If they are compared with the conventional version of the filter [3], that is assuming that the noise is white, the accuracy of the filter improves. In Figure 2b, it can be seen how the innovation sequence presents a more uniform distribution of frequencies. In the conventional version, the innovation has a lack of low frequencies, meaning that it is not purely white. On the other hand, when estimating the system noise, the innovation sequence presents all frequencies at a similar level, as if it were white noise.



(a) Comparison of the estimated force in the degree-of-freedom.

(b) Frequencies histogram comparison of the innovation.

It can be concluded that improving the modeling of the system noise leads to higher accuracy of the estimations in Kalman filtering, and also the behavior of adaptive Kalman filters, where the innovation sequence must be white noise. This approach should be explored in complex systems, where it is expected to obtain greater improvements.

References

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