

## Optimal design and control of the steering of a tilting tricycle.

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### 1 INTRODUCTION

The design of new vehicles is a challenge that benefits from simulation techniques in order to get valid designs before testing, being this a standard in mature industries like automotive or railway. More advanced techniques like kinematic or dynamic optimal design can provide further improvements in early designs making them optimal from some point of view, by calculating the optimal parameters of the design for a given objective function. Optimal control is a similar problem to optimal design, consisting in optimizing some time-dependent control function in order to achieve a prescribed motion or reach an optimal condition. The approach taken in this work is to choose control functions dependent on some constant parameters, therefore the same theoretical framework developed for optimal design remains valid for optimal control also.

The steering optimal design of the tadpole tilting three wheeled vehicle multibody model shown in Figure 1 is not an easy task. The steering system should satisfy Ackerman's steering condition, not only for null roll angles (the typical design for a car steering) but also for any combination of roll and steering angles. Moreover, the relation between the handlebar rotation and the wheels angles should be adjusted. In case we wish to mimic a standard bicycle behavior, this relation must be approximately equivalent to the single handlebar-wheel mount of a common bicycle.

In this work, the optimization of the tilting tricycle, paying especial attention to the steering system, is addressed. Several optimization problems are solved: first the kinematic design optimization of the steering; second the dynamic optimization of the steering, equivalent to the kinematic optimization but solved under dynamic conditions, making possible to design the system to real-drive situations; third, the optimal design of the system, which can be used to program some maneuvers for the dynamical design optimization. All the optimizations performed are gradient-based, they are solved under the same general framework and rely on the multibody sensitivity equations using two approaches: direct sensitivity for optimal design and adjoint sensitivity for optimal control. The approach proposed is general and valid for any multibody system since the starting point for sensitivities are the general kinematics and dynamics equations. The implementation of the sensitivity equations and the numerical experiments were built in the MBSLIM multibody library.

### 2 Optimization and optimal control problem statement

Let us consider a set of objective functions,  $\boldsymbol{\psi} \in \mathbb{R}^o$ , expressed as integrals in time:

$$\boldsymbol{\psi} = \int_{t_0}^{t_f} \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}, \boldsymbol{\lambda}, \boldsymbol{\rho}) dt. \quad (1)$$

The sensitivity analysis of the objective functions with respect to the set of parameters  $\boldsymbol{\rho} \in \mathbb{R}^p$  can be computed by means of direct sensitivity or adjoint sensitivity methods and using the kinematic or the dynamic equations presented before [1, 2].

The parameters  $\boldsymbol{\rho} \in \mathbb{R}^p$  can be any constant magnitude or coefficient affecting the forces of the system, the masses or mass geometry and/or the vector of constraints.

### 3 Numerical experiments

The case study for optimal design and optimal control is the tilting three wheeled vehicle shown in Figure 1. The optimal design can be accomplished by means of a kinematic analysis in positions or by means of a dynamic analysis in order to better optimize for the service conditions of the vehicle.

The objective functions considered enforce the satisfaction of Ackerman's steering principle and the relation between the handlebar rotation and the effective steering angle. For the dynamic simulation, the degrees of freedom of the vehicle are predetermined and the optimization is carried out over this prescribed motion, but for the dynamic simulation, an optimal control function will be added to force the vehicle to fit the desired trajectory and speed, controlling the handlebar and pedals.

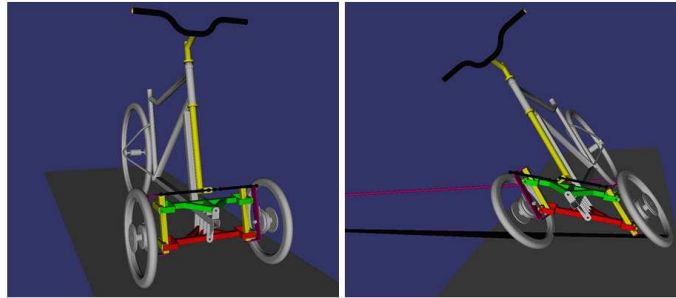


Figure 1: Three wheeled tilting vehicle.

### 4 Conclusions

The present work proves that the approach proposed is a valid approach to improve the design of mechanical systems using kinematics or dynamics simulations. Moreover, the optimal control is also considered and both types of problems can be solved together under the same framework.

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