Optimal trajectory tracking techniques for single-track vehicles

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EXTENDED ABSTRACT

1 Introduction

During the last few years, a wide variety of green urban mobility solutions has emerged around the world. Among them, single-track vehicles have crowded our cities as an affordable, environmentally-friendly and easy-to-use transport. Different types of traditional and electric bicycles, electric scooters and similar vehicles have become more popular, and they are continuously incorporating new functionalities.

In parallel, the autonomous vehicle has also attracted many researchers and companies for its development, including detection of obstacles, identification of trafic signals, interaction with other vehicles, etc. However, most of the efforts have been centered on vehicles with four or more wheels, which are stable with respect to rollover for the velocities and lateral slopes usually considered.

On the contrary, single-track vehicles are commonly unstable in this aspect, and they require the ability of the pilot to avoid their tilting and fall. In this work, different techniques for the optimal trajectory tracking of this type of vehicles is considered without including additional control techniques to avoid the fall of the vehicle.

2 Problem statement

For the sake of clarity, let us consider the multibody model of a simplified bicycle composed of four bodies according to figure 1. It is comprises the frame (B), the fork (H) and the front and rear wheels (F,R), with the dynamic parameters specified in [1]. In this case, instead of non-holonomic constraints, contact-frictional tire forces have been used.

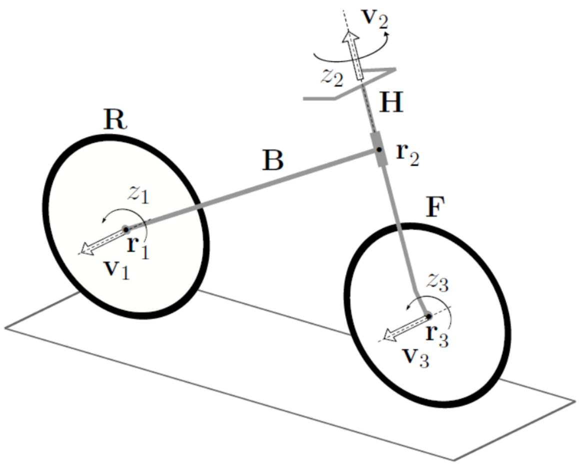

Figure 1: Multibody model of the bicycle

The optimal trajectory tracking problem consists in determining the optimal values of two external torques applied on the rear wheel and on the handlebar, replicating the actions of the pilot. In this work, splines are used to model these external actions over time, reducing the number of optimization parameters while guaranteeing smoothness over time. The parameters considered are torque values at different time steps spaced uniformly over time.

The dynamics of this vehicle is evaluated by means of the global Matrix R formulation described in [2] and summarized here for a system described by $\mathbf{q} \in \mathbb{R}^n$ dependent coordinates, subjected to m constraint equations with $d \geq (n-m)$ degrees of freedom $\mathbf{z} \in \mathbb{R}^d$:

$$\left(\mathbf{R}^{\mathrm{T}}\mathbf{M}\mathbf{R}\right)\ddot{\mathbf{z}} = \mathbf{R}^{\mathrm{T}}\left(\mathbf{Q} - \mathbf{M}\mathbf{S}\mathbf{c}\right), \tag{1}$$

where $\mathbf{M}(\mathbf{q}) \in \mathbb{R}^{n\times n}$ and $\mathbf{Q}(\mathbf{q},\dot{\mathbf{q}},t,\boldsymbol{\rho}) \in \mathbb{R}^n$ are the mass matrix and vector of generalized forces in dependent coordinates, $\mathbf{R} \in \mathbb{R}^{n\times d}$ is a basis of the allowable motions of the kinematic homogeneous system and $\mathbf{Sc} \in \mathbb{R}^n$ is a particular solution of the kinematic acceleration problem with the accelerations of all de degrees set to zero. The vector $\boldsymbol{\rho} \in \mathbb{R}^p$ represent the parameters of the system.

Gradient-based optimization algorithms are considered for the optimal trajectory problem, thus an accurate and efficient sensitivity analysis of the dynamic response of the system given by equations (1) is also required. For that purpose, the Matrix R adjoint sensitivity formulation in natural coordinates described in [3] have been used to obtain the solutions presented. According to the reference, the adjoint variable method is better suited for sensitivity problems involving a large number of parameters and a few objective functions.

3 Optimal trajectory tracking

The objective function can be defined in this optimization problem as:

$$\Psi = \int_{t_0}^{t_F} ((r_{3x} - r_{ex})^2 + (r_{3y} - r_{ey})^2)\mathrm{d}t \tag{2}$$

wherein r_{3x} and r_{3y} are the X and Y components of $\mathbf{r}_3$, while $r_{ex} = r_{ex}(t)$ and $r_{ey} = r_{ey}(t)$ are the X and Y coordinates of the desired trajectory, respectively. The integral limits are $t_F = 12\,s$. and $t_0 = 0\,s$.

In this problem, the parameters of the system are the values of the steering and throttle torques over the handlebar and rear wheel, respectively. The result of the dynamic optimization is the optimal throttle and steering which makes (2) minimal.

4 Results and conclusions

The initial trajectory guess (a straight maneuver), the desired (reference) trajectory and the optimized trajectory are presented in figure 2, showing that the method proposed is able to follow the desired trajectory from an initial guess far from the optimal controls. It is important to remark that no additional conditions to maintain the stability of the vehicle are needed, the optimal control which minimizes (2) is able to follow the trajectory keeping the stability of the bicycle because if the bicycle falls the solution obtained would not be optimal therefore the optimization process leads to a stable maneuver without the need for any explicit controller to keep the leaning angle bounded.

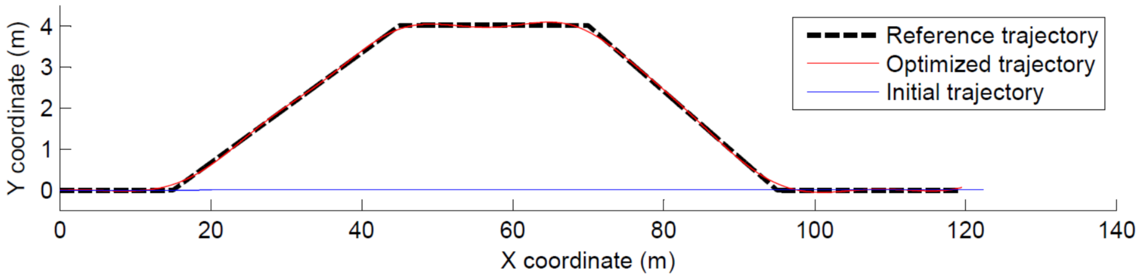

Figure 2: Initial and optimized trajectories.

Acknowledgments

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