Analytical sensitivity analysis of flexible multibody systems composed by rigid bodies and beams.

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Abstract

Optimization of the dynamics of multibody systems is an active area of research with many important applications in different fields. Among many available optimization techniques, gradient methods are very versatile and popular; and one of its main ingredients is the computation of sensitivities. The point of departure is an objetive functional ψ, with the general expression:

$$\psi = \int_{t_0}^{t_F} g(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\rho}) \ \mathrm{dt} \tag{1}$$

$\boldsymbol{\rho}$ being a vector containing the parameters of the model. This problem is typically solved employing a gradient method, that requires the computation of the gradient of the funcional respect the parameters; aplying the chain rule:

$$D_{\boldsymbol{\rho}}\psi = \int_{t_0}^{t_F} \left[(D_{\mathbf{q}}g)(D_{\boldsymbol{\rho}}\mathbf{q}) + (D_{\dot{\mathbf{q}}}g)(D_{\boldsymbol{\rho}}\dot{\mathbf{q}}) + D_{\boldsymbol{\rho}}g\right] \ \mathrm{dt} \tag{2}$$

The sensitivities $D_{\boldsymbol{\rho}}\mathbf{q}$ and $D_{\boldsymbol{\rho}}\dot{\mathbf{q}}$ provides information about how the coordinates of the system change with time when the parameters change. Multibody systems are typically represented by systems of nonlinear differential equations:

$$\mathbf{M}\ddot{\mathbf{q}} + D_{\mathbf{q}}\Phi^{\mathrm{T}}\alpha\Phi = \mathbf{Q} \tag{3}$$

where it has been considered that the system is subjected to holonomic constraints, collected in vector Φ, that are enforced by a penalty method. Sensitivities are then computed evaluating the corresponding derivatives respect the parameters around the reference movement. These derivatives (sensitivities) depends on time and are the solutions of a system of linear differential equations (with variable coefficients) of the form:

$$\mathbf{M}\ddot{\mathbf{q}}' + \hat{\mathbf{K}}\mathbf{q}' + \hat{\mathbf{Q}} = \mathbf{0} \tag{4}$$

where the notation $(\cdot)' = D_{\boldsymbol{\rho}}(\cdot) = \partial(\cdot)/\partial\boldsymbol{\rho}$ has been employed; thus, $\mathbf{q}', \dot{\mathbf{q}}'$ and $\ddot{\mathbf{q}}'$ are the position, velocity and acceleration sensitivities respectively. The terms $\hat{\mathbf{K}}$ and $\hat{\mathbf{Q}}$ are:

$$\hat{\mathbf{Q}} = \mathbf{Q}' + D_{\mathbf{q}}\boldsymbol{\Phi}^{T'}\alpha\boldsymbol{\Phi} + D_{\mathbf{q}}\boldsymbol{\Phi}^{T}\alpha\boldsymbol{\Phi}' + \mathbf{M}'\ddot{\mathbf{q}} \tag{5}$$

$$\hat{\mathbf{K}} = D_{\mathbf{q}}\mathbf{Q} + D_{\mathbf{qq}}^{\mathrm{T}}\alpha\boldsymbol{\Phi} + D_{\mathbf{q}}\boldsymbol{\Phi}^{T}\alpha D_{\mathbf{q}}\boldsymbol{\Phi} \quad , \tag{6}$$

Their computation may be performed after the solution for the dynamics, or simultaneously with it.

Sensitivity analysis of mechanisms exclusively composed by rigid bodies has been studied in many works of the literature [1]. However, analysis dealing with flexible mechanisms are rarer. In this work we show the results of a sensitivity analysis of special systems, where the flexible parts are slender beams, that could be represented either by a nonlinear beam model [2] or with solid elements.

The specific nonlinear beam model considered in this work is defined as a collection of n identical

deformable 1D segments (only withstand tensile or compressive forces) with regular section moving in a three-dimensional Euclidean space. Each segment is defined by two nodes, and two consecutive segments share one node. The axial response of the beam is represented by the deformation of the segments and is governed by an hyperelastic potential. The bending response is represented by the misalignment of consecutive trusses and is governed by another potential. The sum of both potentials is an approximation of the strain energy of the beam.
The approach with solid elements will explore the possibilities of performing the analytical derivatives in the isoparametric formulation of linear tetrahedra and/or bilinear hexahedra defining a beam. In both cases (beam or solid elements), the relevant proposal of this work is the analytical deduction of the sensitivities associated with the deformable parts, that are expected to improve the efficiency and accuracy of the computations.

A simple and physically intuitive approach based on a finite-difference method is used for validating the preliminary sensitivity results Some simple numerical examples are presented showing the performance of the proposed formulation.

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References

[1] D. Dopico, Y. Zhu, A. Sandu and C. Sandu: Direct and adjoint sensitivity analysis of ordinary differential equation multibody formulations. J. Comp. And Nonlinear Dynamics No. 10, 2015.

[2] J.C. García: A Simple Shear and Torsion-free Beam Model for Multibody Dynamics. J. Comp. And Nonlinear Dynamics No. 5, pp. 1-8, 2017