Inferring Subsystem Dynamics from Data In Explicit Co-Simulation of Mechanical Systems

Maciej Pikuliński*, **Paweł Malczyk***, **Antonio J. Rodríguez**†, **Francisco González**†

* Division of Theory of Machines and Robots
Institute of Aeronautics and Applied Mechanics
Faculty of Power and Aeronautical Engineering
Warsaw University of Technology
Nowowiejska 24, 00-665 Warsaw, Poland
{maciej.pikulinski.dokt, pawel.malczyk}@pw.edu.pl

† Laboratorio de Ingeniería Mecánica
Campus Industrial Ferrol - CITENI
Universidade da Coruña
Campus de Esteiro, 15403, Ferrol, Spain
{antonio.rodriguez.gonzalez, f.gonzalez}@udc.es

1 Introduction

Co-simulation environments are simulation setups in which two or more dynamic solvers are coupled by exchanging of a limited set of variables at specific communication points in time. The integration of the dynamics of each solver during a macro-step, i.e., between two consecutive communication points, proceeds without further input from the rest of the environment, which increases the modularity of the simulation solution but, at the same time, makes it necessary to coordinate the different subsystems. This coordination can be achieved repeating the integration of the dynamics of each subsystem between communication points until convergence is reached. In some applications, however, it is impossible to follow this approach, because some subsystems cannot reset their internal state and go back to a past instant in time to restart their integration, or because the time available to carry out numerical computations is limited. This is the case, for instance, of cyber-physical devices, in which at least one subsystem is a hardware device. In these situations, *explicit* co-simulation schemes, in which macro-steps are evaluated only once, are mandatory.

2 Problem statement

Explicit co-simulation schemes suffer from accuracy and stability issues that stem from the discontinuities and delays introduced by the discrete-time co-simulation interface. They alter the system energy and give rise to high-frequency disturbances in its overall dynamics [2]. The approaches proposed to address these issues often rely on correcting of the coupling variables to provide a more faithful description of the dynamics. This can be done by means of signal reconstruction through polynomial extrapolation or the implementation of control algorithms. Another possibility is providing critical subsystems with a prediction of the evolution of the dynamics of others using physics-based solutions, such as reduced-order descriptions of the interface between subsystems [1]. This is an effective way to alleviate the issues of explicit co-simulation, but it requires the knowledge of the equations that govern the dynamics of other subsystems in the simulation setup. Often, these are unavailable, because co-simulation solvers are frequently designed to be *black boxes* and the only information they share with their environment is conveyed in their coupling variables.

3 Contribution and results

In this paper, we provide a data-driven framework to infer subsystem dynamics in explicit co-simulation environments to make the co-simulation much less vulnerable to errors in total energy. Specifically, we use Dynamic Mode Decomposition (DMD) [3] to extract local-linear models within one subsystem out of the other subsystem input-output measurements. The DMD technique extracts the relationships between pairs of state measurement data $\mathbf{x}$ and input (actuation) signals $\mathbf{u}$. The discrete linear subsystem dynamics can be generalized for data snapshots collected in time as follows:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k \quad \rightarrow \quad \mathbf{X}' = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{U}, \quad \text{where}$$
$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_{m-1} \\ | & | & & | \end{bmatrix}, \mathbf{X}' = \begin{bmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \dots & \mathbf{x}_m \\ | & | & & | \end{bmatrix}, \mathbf{U} = \begin{bmatrix} | & | & & | \\ \mathbf{u}_1 & \mathbf{u}_2 & \dots & \mathbf{u}_{m-1} \\ | & | & & | \end{bmatrix}, \tag{1}$$

where m is the total number of snapshots and $\mathbf{X}$, $\mathbf{X}'$, and $\mathbf{U}$ are snapshot matrices. The DMD is focused on finding best-fit approximations to the unknown subsystem matrices $\mathbf{A}$ and $\mathbf{B}$ out of a sequence of state and control inputs [3], [4]. A least-squares solution can be found by minimizing the Frobenius norm of $\left\| \mathbf{X}' - \begin{bmatrix} \mathbf{A} & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{U} \end{bmatrix} \right\|_F$, to get $\begin{bmatrix} \mathbf{A} & \mathbf{B} \end{bmatrix} = \mathbf{X}' \begin{bmatrix} \mathbf{X} \\ \mathbf{U} \end{bmatrix}^\dagger$. The employed data-driven method requires only snapshots of state and actuation data collected in the co-simulation. It helps to keep the numerical errors resulting from the co-simulation under control. We demonstrate the proposed approach on a simple linear oscillator (see fig. 1) arranged following a force-displacement coupling scheme [1].

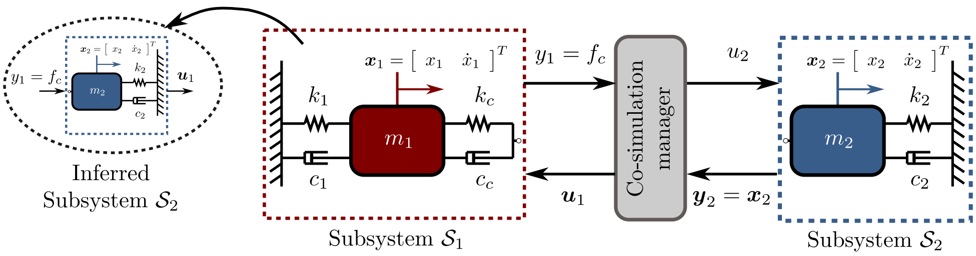

Figure 1: A two-degree-of-freedom linear oscillator, $m_1 = m_2 = 1\,\mathrm{kg}$, $k_1 = 10\,\frac{\mathrm{N}}{\mathrm{m}}$, $k_2 = 1000\,\frac{\mathrm{N}}{\mathrm{m}}$, $k_c = 100\,\frac{\mathrm{N}}{\mathrm{m}}$, $c_1 = c_2 = c_c = 0\,\frac{\mathrm{Ns}}{\mathrm{m}}$ (force-displacement coupling scheme).

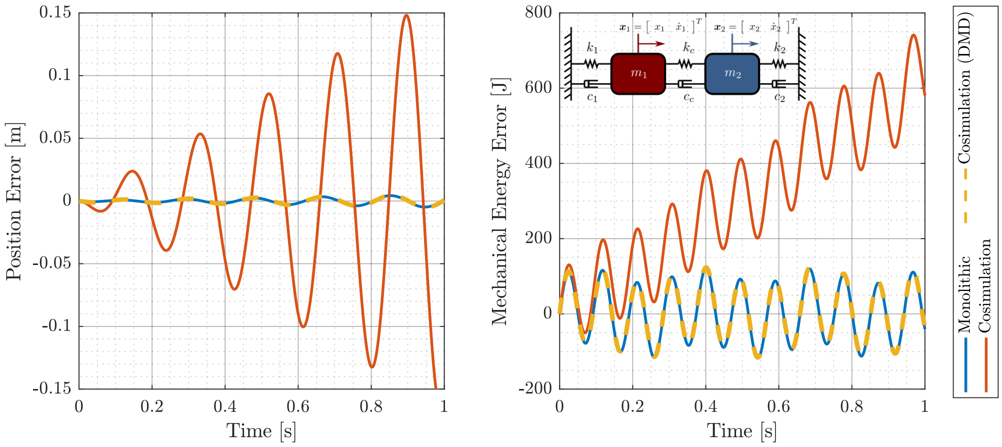

Figure 2: Comparison of Position Error (left) and Mechanical Energy Error (right) relative to the analytical solution for Monolithic solver, Cosimulation, and Cosimulation with DMD.

Several numerical experiments are performed to investigate the validity of the approach proposed herein. Preliminary results are shown in fig. 2. Explicit co-simulation leads to spurious energy artifacts at the discrete-time interface between subsystems. The resulting energy errors deteriorate the accuracy of the co-simulation results and may, in some cases, develop into the instability of the numerical integration process. The DMD method discovers the dynamics of subsystem 2 within subsystem 1 and improves the state estimates of subsystem 2 by predicting those values from the inferred linear model. Then, the coupling force f_c is corrected and sent to subsystem 2, resulting in the overall improvement of the total energy conservation over time and position errors.

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