

Sensitivity Analysis of the FFR ALI3-P Flexible Multibody Formulation

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1 Introduction

In many engineering applications, the effects of the elastic deformation of bodies are not negligible and have a significant influence on the overall dynamics of a machine. In the last few decades, an important effort has been devoted to the development of models and formulations for the accurate and efficient simulation of flexible multibody systems, but only recently the sensitivity analysis of these formulations has been explored in such depth, with a few prominent exceptions [1, 2].

The flexibility of a body can be parameterized in different forms. The most extended option in the multibody community is the Floating Frame of Reference (FFR) method [4], in which rigid and flexible coordinates coexist in a single body: the rigid body coordinates describe the rigid body motion of a reference frame, while the flexible coordinates measure the elastic deformation with respect to a local undeformed configuration. Assuming small deformations, the position of any point can be calculated by superposition of the flexible deformation over the rigid body motion.

The FFR method is frequently combined with model order reduction techniques in order to minimize the number of flexible coordinates while keeping an acceptable deformation profile. The selection of deformation modes is a critical stage in the parameterization of a flexible body and has a direct impact on the quality of the results and on the efficiency of the simulation [6].

Constraints can be enforced in flexible multibody systems modeled with FFR by means of any of the well known methods employed in rigid multibody systems. Among them, the FFR Augmented Lagrangian Index-3 formulation with velocity and acceleration projections (FFR ALI3-P) in natural coordinates described in [3] has evidenced an excellent balance between accuracy and efficiency.

The purpose of this work is the study of the sensitivity analysis of the FFR ALI3-P formulation using the direct differentiation method. Analytical differentiation has been used for the core derivatives (wrt states), but the calculation is open to analytical and numerical differentiation for partial derivatives with respect to the parameters. This sensitivity formulation has been implemented in the general purpose multibody library MBSLIM [5] which now supports kinematics, dynamics, sensitivity analysis and optimization of rigid and flexible multibody systems.

2 Dynamic Multibody Formulation

The first step in a FFR dynamic formulation when modal reduction is used consists in a preprocess where the invariant inertial shape integrals are evaluated from the results of a modal analysis or other calculations which provide suitable deformation modes.

According to [3], the dynamics of a flexible multibody system can be calculated by means of a three-stage process. The first step consists in an Augmented Lagrangian index-3 problem of the form:

$$[\mathbf{M}\ddot{\mathbf{q}}^* + \mathbf{K}\mathbf{q} + \Phi_{\mathbf{q}}^T(\boldsymbol{\lambda}^* + \boldsymbol{\alpha}\Phi)]^{\{i\}} = \left[\mathbf{Q}_e - \mathbf{M}\dot{\mathbf{q}}^* + \frac{1}{2}(\dot{\mathbf{q}}^{*\text{T}}\mathbf{M}_{\mathbf{q}}\dot{\mathbf{q}}^*)^{\text{T}} \right]^{\{i\}} \quad (1a)$$

$$\boldsymbol{\lambda}^{*\{i+1\}} = \boldsymbol{\lambda}^{*\{i\}} + \boldsymbol{\alpha}\Phi^{\{i\}}; \quad i = 0, 1, 2, \dots \quad (1b)$$

being \mathbf{q} the set of rigid and flexible coordinates, Φ the vector of kinematic constraints, \mathbf{M} the mass matrix, \mathbf{Q}_e the vector of external forces, \mathbf{K} the stiffness matrix accounting for flexibility, $\dot{\mathbf{q}}^*$ the unprojected velocities, $\ddot{\mathbf{q}}^*$ the unprojected accelerations, $\boldsymbol{\lambda}$ the approximate Lagrange multipliers, $\boldsymbol{\alpha}$ a diagonal penalty matrix and the superscript i indicates the iteration index.

Equations (1) can be stabilized through projections onto the velocity and acceleration constraint manifolds with:

$$(\bar{\mathbf{P}} + \zeta \Phi_q^T \alpha \Phi_q) \dot{\mathbf{q}} = \bar{\mathbf{P}} \dot{\mathbf{q}}^* - \Phi_q^T \zeta \alpha \Phi_t \quad (2)$$

$$(\bar{\mathbf{P}} + \zeta \Phi_q^T \alpha \Phi_q) \ddot{\mathbf{q}} = \bar{\mathbf{P}} \ddot{\mathbf{q}}^* - \Phi_q^T \zeta \alpha (\dot{\Phi}_q \dot{\mathbf{q}} + \ddot{\Phi}_t) \quad (3)$$

where $\bar{\mathbf{P}}$ is a symmetric projection matrix and ζ is a penalty factor.

3 Sensitivity Analysis and numerical experiments

The sensitivity analysis of the FFR ALI3-P formulation is addressed by means of the direct differentiation method, which requires the solution of as many systems of equations as sensitivity parameters. The sensitivity equations are not presented here due to lack of space.

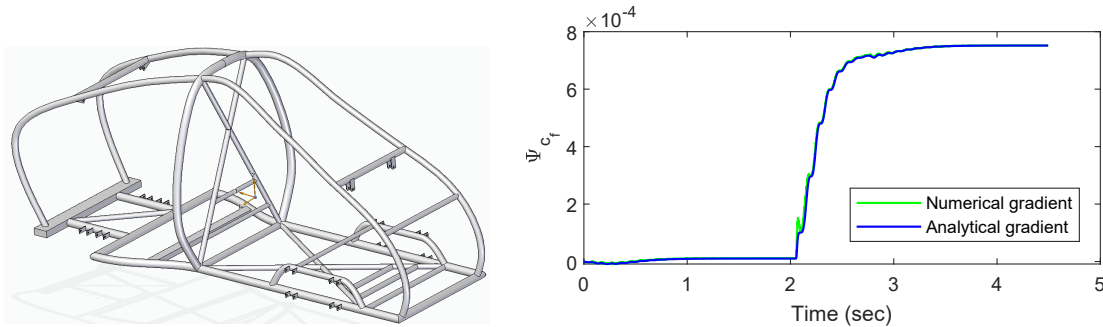


Figure 1: Flexible chassis of the vehicle simulated (left) and objective function gradient (right).

The MBSLIM implementation of the sensitivity expressions have been tested in different benchmark problems and in a more complex vehicle, in which the elastic deformation of the chassis (displayed in figure 1) is monitored. The performance of the sensitivity formulation is assessed in terms of accuracy and efficiency against numerical differentiation (figure 1).

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