Practical computation of initial sensitivities in dynamics of flexible multibody systems

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EXTENDED ABSTRACT

1 Introduction

Sensitivities appear in practice in different situations while analyzing the dynamics of multibody systems [1]. One situation of interest arises simply by the desire of evaluating how a specific coordinate of the system is influenced by a variation of one of the parameters, which is the direct definition of sensitivity. In a more general context, sensitivities appear when using gradient methods to perform an optimization analysis, searching for the extreme of objective functionals of the form $\psi = \int_{t_0}^{t_F} g(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\rho})\ \mathrm{dt}$, where $\mathbf{q} \in \mathbb{R}^n$ and $\dot{\mathbf{q}} \in \mathbb{R}^n$ are vectors collecting generalized coordinates and velocities respectively, and $\boldsymbol{\rho} \in \mathbb{R}^p$ parameters of the model. In this case, using the notation $D_\square(\cdot) = \partial(\cdot)/\partial\square$ and $\dot{(\cdot)} = D_t(\cdot) = \partial(\cdot)/\partial t$, sensitivities $(D_{\boldsymbol{\rho}}\mathbf{q})$ and $(D_{\boldsymbol{\rho}}\dot{\mathbf{q}})$ appear when applying the chain rule to the gradient of the aforementioned functional:

$$D_{\boldsymbol{\rho}}\psi = \int_{t_0}^{t_F} \left[(D_{\mathbf{q}}g)(D_{\boldsymbol{\rho}}\mathbf{q}) + (D_{\dot{\mathbf{q}}}g)(D_{\boldsymbol{\rho}}\dot{\mathbf{q}}) + D_{\boldsymbol{\rho}}g\right]\ \mathrm{dt} \tag{1}$$

On the other hand, dynamics of multibody systems are typically represented by a set of index-3 algebraic-differential equations (DAE):

$$\mathbf{M}\ddot{\mathbf{q}} + (D_{\mathbf{q}}\boldsymbol{\Phi})^{\mathrm{T}}\boldsymbol{\lambda} = \mathbf{Q} \quad , \quad \boldsymbol{\Phi} = \mathbf{0} \tag{2}$$

$\mathbf{M} \in \mathbb{R}^{n\times n}$ being the mass matrix, $\boldsymbol{\Phi} \in \mathbb{R}^m$ a vector collecting holonomic constraints, $\boldsymbol{\lambda} \in \mathbb{R}^m$ the vector of Lagrange multipliers and $\mathbf{Q} \in \mathbb{R}^n$ the active forces. Other constraint-enforcing methods, such as penalty or augmented Lagrange, would lead to different formulations, but any choice is irrelevant for the purpose of this study. Sensitivities are then computed evaluating the corresponding derivatives respect the parameters around the reference movement. These derivatives are functions of time and are the solutions of a system of linear differential equations (with variable coefficients) named Tangent Linear Model (TLM):

$$\mathbf{M}\ddot{\mathbf{q}}' + \hat{\mathbf{K}}\mathbf{q}' + (D_{\mathbf{q}}\boldsymbol{\Phi})^{\mathrm{T}}\boldsymbol{\lambda}' = \hat{\mathbf{Q}} \quad , \quad (D_{\mathbf{q}}\boldsymbol{\Phi})^{\mathrm{T}}\mathbf{q}' + \boldsymbol{\Phi}' = \mathbf{0} \tag{3}$$

where the notation $(\cdot)' = D_{\boldsymbol{\rho}}(\cdot) = \partial(\cdot)/\partial\boldsymbol{\rho}$ has been employed; thus, $\mathbf{q}', \dot{\mathbf{q}}', \ddot{\mathbf{q}}'$ and $\boldsymbol{\lambda}'$ are the position, velocity, acceleration and Lagrange multiplier's sensitivities respectively. The terms $\hat{\mathbf{K}}$ and $\hat{\mathbf{Q}}$ depend on $\mathbf{q}, \dot{\mathbf{q}}$ and $\boldsymbol{\lambda}$ of the reference movement.

2 Initial sensitivities

The TLM (3), that can be solved in advance or simultaneously with this reference movement [2], must be complemented with a set of initial conditions $\mathbf{q}'_0$ and $\dot{\mathbf{q}}'_0$, that have to satisfy the constraints:

$$\left.\frac{\mathrm{d}\boldsymbol{\Phi}}{\mathrm{d}\boldsymbol{\rho}}\right|_0 = \mathbf{0} \quad \rightarrow \quad (D_{\mathbf{q}}\boldsymbol{\Phi})_0\mathbf{q}'_0 = -\boldsymbol{\Phi}'_0 \tag{4}$$

$$\left.\frac{\mathrm{d}\dot{\boldsymbol{\Phi}}}{\mathrm{d}\boldsymbol{\rho}}\right|_0 = \mathbf{0} \quad \rightarrow \quad (D_{\mathbf{q}}\boldsymbol{\Phi})_0\dot{\mathbf{q}}'_0 = -\left\{\left[(D_{\mathbf{qq}}\boldsymbol{\Phi})\dot{\mathbf{q}} + (D_{\mathbf{q}}\dot{\boldsymbol{\Phi}})\right]\mathbf{q}' + (D_{\mathbf{q}}\boldsymbol{\Phi})'\dot{\mathbf{q}} + (\dot{\boldsymbol{\Phi}})'\right\}_0 \tag{5}$$

where the subscript 0 denotes evaluation at $t = 0$. Systems (4) and (5) have m equations for n unknowns each; so, assuming that the constraints are not redundant, each has to be complemented identifying $n - m$ coordinates as degrees of freedom (DOF) and assign to them given initial sensitivities. Their specific values are defined by the user and depend on the perturbed case to be analyzed.

Apart from those related with kinematic constraints representing joints with other bodies, flexible bodies do not have constraints. Focusing in a flexible body discretized with a Finite Element Method for practical purposes, it will have as many DOFs as the number of nodes multiplied by the number of (generalized) coordinates of each node, minus the number of coordinates involved in the joint constraints. The criterion for assigning initial sensitivities to these DOFs is usually scarcely explained in the literature. Sometimes the problem is circumvented using automatic or numerical differentiation, and sometimes the explanation is regarded as implementation details that are not worth to describe. While this may look true at first sight, the experience of the authors have been very frustrating while dealing with these computational overlooked details, even for simple problems. What is more, their impact on the results is so great that we have considered it deserves some attention.

The simple proposal presented in this work is to assume that, for the initial sensitivities' computation, the flexible body is rigid. This approach has been already employed in the literature, but the specific way of doing it could be diverse. In this proposal, a set of constraints are added that are active just at $t = 0$, and disappears afterwards. While the idea is extremely simple, the details of the formulation for different type of flexible bodies (line-like with beams [3] or 2D and 3D solid with solid elements) are worth to explore, and the implementation could be challenging, depending on the general multibody formulation of the software.

A description of how specific constraints can be added for each situation, along with results of their implementation for 3D flexible multibody systems, incorporating beam and 3D solid elements will be provided. The conclusion is that the methodology is sufficiently general, simply and robust to be considered in practical computations with complex systems.

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