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Efficient simulations of multibody systems with clearances using reduced interface modelling

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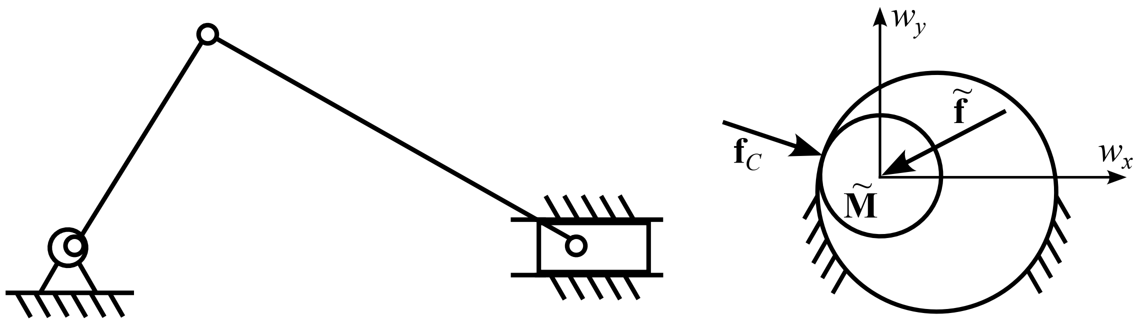
Mechanical systems with unlubricated clearance joints are common in various engineering applications. Clearances, coming from manufacturing tolerances, assembly errors, or wear, cause intermittent contact and impacts. Simulating the forward dynamics of multibody systems, which exhibit such non-linear phenomena, is generally computationally demanding task. Traditional approach to modelling of rigid-body impacts lies basically in application of constitutive equations and penalty methods [4], leading to stiff numerical problems and, consequently, small integration step sizes. Recently, alternative formulation as a problem of non-smooth dynamics has been increasingly used to approach such calculations. This practise results in complementarity problems as discussed in [1]. Unfortunately, both ways are computationally quite expensive even in cases of simple mechanical systems such as a slider-crank mechanism with one clearance joint, which is visualized in Fig. 1a.



(a) Slider-crank mechanism (b) Reduced interface model

Fig. 1. Schemes of the slider-crank mechanism and the reduced interface model

The study presented in this paper is based on the penalty approach. A general planar multibody problem can be formulated in the matrix notation as

$$\begin{bmatrix} \mathbf{M} & \partial_{\mathbf{q}}\mathbf{c}^{\mathrm{T}} \\ \partial_{\mathbf{q}}\mathbf{c} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_E + \mathbf{f}_C \\ \mathbf{f}_D \end{bmatrix}, \tag{1}$$

where $\mathbf{q}$ is the vector of generalized coordinates, $\mathbf{M}$ stands for the mass matrix, $\mathbf{c}$ is the vector of kinematic constraints defining the Jacobian matrix $\partial_{\mathbf{q}}\mathbf{c}$, $\mathbf{f}_E$ contains external forces, $\mathbf{f}_D$ is the term quadratic in velocities resulting from the double time-differentiation of constraint equations $\mathbf{c}(\mathbf{q}) = \mathbf{0}$, $\mathbf{f}_C$ is the vector of contact forces, and $\boldsymbol{\lambda}$ represents the vector of Lagrange multipliers. The contact force $\mathbf{f}_C$ acts as a penalty force generated when contact in the clearance joint is present. The simplest non-linear relation called the Hertz model expresses the normal contact force as

$$f_N = k\,\delta^n, \tag{2}$$

where k is the contact stiffness usually determined using the Hertzian theory again, and the exponent n is mostly equal to $3/2$ in the case of parabolic distribution of contact stresses according to [2]. High value of the contact stiffness results in the necessity of very small integration step size.

Model (1) with contact forces is used for high-fidelity prediction of the imperfect system's dynamics, including precise determination of relative movements inside imperfect joints. In the case all joints include no or negligible clearance, the system can be modelled only using kinematic constraints (i.e., perfect joints). Thus, it does not include any contact forces and $\mathbf{f}_C$ is zeroed in (1). Such ideal system can be simulated with efficient multibody algorithms that avoid the need for small integration steps.

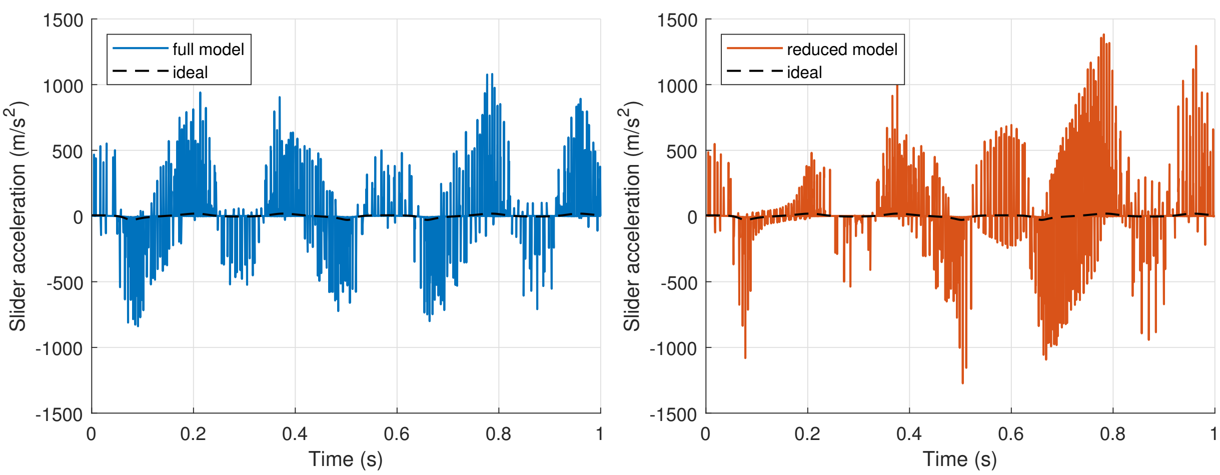
This work proposes methodology, the so-called reduced interface model, that enables reliable capturing of the imperfect system's dynamics with significantly decreased computational effort. The study uses the idea that imperfect system's states (position and velocity) can be approximated by states of the ideal system, which is a completely valid assumption for systems with relatively small clearances. Based on [3], these states are used for calculation of the effective mass matrix $\tilde{\mathbf{M}}$ and the effective force vector $\tilde{\mathbf{f}}$, which are utilized in the reduced interface model

$$\tilde{\mathbf{M}}\dot{\mathbf{w}} = \tilde{\mathbf{f}} + \mathbf{f}_C \tag{3}$$

that captures the relative dynamics of the clearance joint when all other mechanical parts of the system are reduced to effective terms $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{f}}$. The vector $\mathbf{w}$ represents the relative velocity of the journal inside the bearing. The described reduced interface model is illustrated in Fig. 1b.

Effective terms $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{f}}$ are evaluated with the time rate equalling integration step size of the ideal system, which is relatively large, e.g., $h_1 = 10^{-3}$ s. Integration step size of the reduced model representing the clearance joint is small enough to capture contact phenomena in the joint, e.g., $h_1 = 10^{-5}$ s.

The methodology is applied on the simulation of the slider-crank mechanism with one clearance joint between the crank and ground depicted in Fig. 1a. Fig. 2 compares the acceleration



(a) Full model

(b) Reduced interface model

Fig. 2. Acceleration of the slider

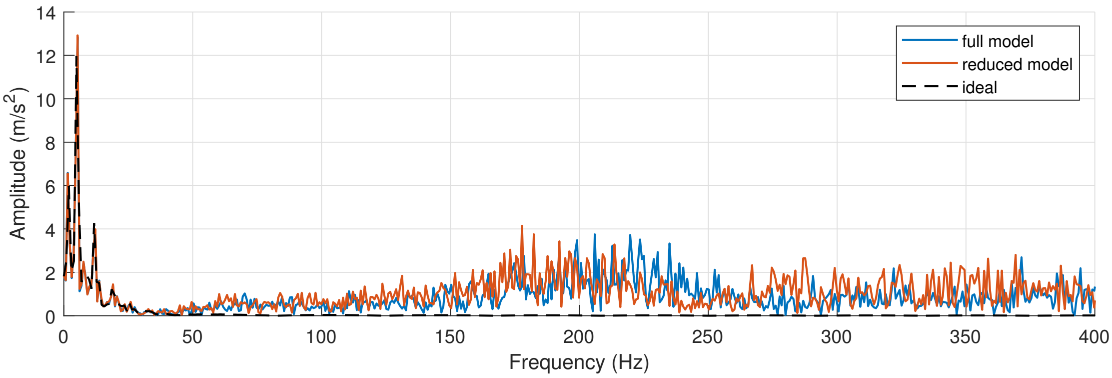


Fig. 3. FFT of slider's acceleration

of the slider obtained by simulation of the full-size multibody model and by simulation of the reduced model. Fig. 3 shows both signals in the frequency domain converted from the time domain using the Fast Fourier Transform. It is evident that the results are not completely identical, nevertheless, reduced model produces quite similar data that correctly capture the influence of the clearance joint on the mechanism's motion. The great benefit of the methodology lies in the fact that the computing time was reduced by more than 85 %. The data coming from the reduced model can be then utilized, e.g., to train neural-networks or to provide a model for state estimation. For these purposes, extreme precision of results is not necessary but since lots of data must be generated, computational efficiency is required.

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