Optimization of flexible multibody systems based on the floating frame of reference with analytical sensitivities

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Abstract

Design optimization based on the dynamics of multibody systems usually requires the sensitivity analysis of the equations of motion, if gradient based optimization methods are selected.

Calculating the sensitivities of the equations of motion is not a trivial task, therefore they are often calculated by means of finite differences, in order to avoid the complexity of dealing with the terms involved in the sensitivity equations, which imply systematically taking derivatives of mass matrices, constraint equations and generalized forces with respect to both states and parameters of the system. Depending on the number of parameters involved, the numerical procedures can be very demanding in terms of computational time and the accuracy obtained can be very poor in many cases. In this work, pure analytical procedures for the sensitivity of flexible multibody systems are explored and other techniques, like numerical perturbations or symbolic expressions, are left as validation techniques.

The sensitivity equations have been systematically derived, in the context of rigid multibody systems, for various formulations of the equations of motion: index-3 DAE [4], index-1 DAE [1], Baumgarte and penalty [7], Matrix-R [2], ALI3-P formulations [3] or augmented Hamiltonian [6]. It turns out that the complexity of calculating all the terms is usually higher than the assembly and integration of the sensitivity equations themselves.

For general multibody systems with complex phenomena, the task of getting the sensitivities by analytical methods can become very complex or even impractical. This is the case when flexible bodies are involved. For complex geometries, most of flexible body methods rely on the finite element method (FEM) in some way and in the case of the Floating Frame of Reference (FFR) formulation, that dependency comes from the stiffness matrix and deformation modes, which are usually a combination of static and dynamic modes calculated by means of a FEM preprocess.

The five-bar mechanism has been used as benchmark experiment in multiple dynamic [5] and sensitivity formulations. This rigid multibody model has been altered to include the flexibility of one of its bars (between points 1 and 2 of figure 1), made of steel ($E = 206.94\times10^{-9}\,\text{Pa}$, $\nu = 0.288$ and $\rho = 7829.0\,\text{kg}/\text{m}^3$) with an initial length of $L_{12} = \frac{1}{2}\sqrt{13}\,\text{m}$ and a rectangular cross-section of $7.29\times10^{-3}\,\text{m}\times14.58\times10^{-3}\,\text{m}$. The orientation of the flexible bar is such that the largest cross-section dimension coincides with the axis of the revolute joints.

The following objective function was chosen:

$$\psi = \int_{t_0}^{t_F} (\mathbf{r}_2 - \mathbf{r}_{20})^{\mathrm{T}} (\mathbf{r}_2 - \mathbf{r}_{20})\, \mathrm{dt} \tag{1}$$

where $\mathbf{r}_2$ is the global position of the point 2 and $\mathbf{r}_{20}$ is the initial position (at t_0) of the same point. As parameters to obtain the sensitivities, the natural lengths of the springs were chosen $\boldsymbol{\rho}^{\mathrm{T}} = [L_{01}, L_{02}]$. This optimization problem has a trivial solution when $\mathbf{r}_2(t) = \mathbf{r}_{20}$ which is satisfied for the static equilibrium position.

Two optimization problems with two different sets of parameters (and two levels of complexity associated) have been solved:

- The first set of optimization parameters chosen includes the natural lengths of the springs $\boldsymbol{\rho}^{\mathrm{T}} = [L_{01}, L_{02}]$. These parameters do not affect the geometry of the flexible body, thus the stiffness

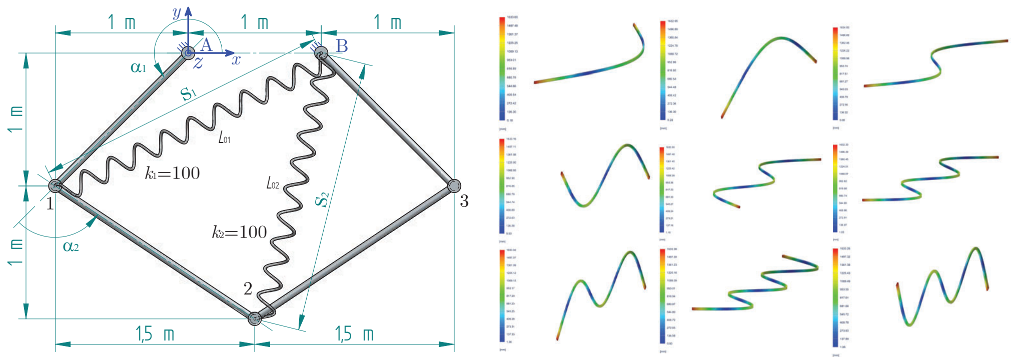

Figure 1: Five-bar mechanism and first 9 deformation modes from a free-free modal analysis of the flexible bar.

matrix and deformation modes are constant during the whole optimization, but yet the sensitivity equations need for the derivatives of the flexible mass matrices, inertial force vectors, flexible body constraints, etc., with respect to the rigid and flexible body states and other parameters.

- The second set of optimization parameters includes the flexible body lenght as well: $\boldsymbol{\rho}^{\mathrm{T}} = [L_{01}, L_{02}, L_{12}]$. The complexity is much higher on this case, because one parameter affects the geometry of the flexible body and therefore, the derivatives of the finite element stiffness matrix and deformation modes, show up in the multibody sensitivity equations.

The sensitivity analysis of the dynamics of the mechanism, considering each set of parameters, is used to compute the gradient of the objective function. For the optimization, an in-house augmented Lagrangian algorithm has been used, proving that the whole set is able to optimize the problem.

Acknowledgments

The support of the Spanish Ministry of Science, Innovation and Universities (MICIU) under projects PID2020-120270GB-C21 and PID2024-158324OB-I00 are greatly acknowledged.

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